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Complexity Theory

SS 2016

Homework 11

Exercise 1 (10 points)**:**
Let $A, B \subseteq \{0,1\}^*$. Show that

- If $A \in \mathbf{P}$, then $\mathbf{P}^A = \mathbf{P}$.
- If $\mathbf{NP} = \mathbf{P}^{SAT}$, then $\mathbf{NP} = \text{co-}\mathbf{NP}$.
- $\overline{SAT} \in \mathbf{P}^{SAT}$.
- If $A \leq_P B$, then $\mathbf{P}^A \subseteq \mathbf{P}^B$.
- There is a $C \subseteq \{0,1\}^*$ s.t. $\mathbf{P} \neq \mathbf{P}^C$.
- If $A \subseteq \{0,1\}^k$ for some $k \in \mathbb{N}$, then $\mathbf{P}^A = \mathbf{P}$.

Exercise 2 (10 points)**:**
Consider the DTM D from the space hierarchy theorem proof:
$D =$ "On input $w \in \{0,1\}^*$:
1. Let n be the length of w.
2. Compute $f(n)$ using space constructibility, and mark off this much tape. If later stages ever attempt to use more space, *reject*.
3. If w is not of the form $\langle M \rangle 10^*$, *reject*.
4. Simulate M on input w while counting the number of steps used in the simulation. If the count ever exceeds $2^{f(n)}$, *reject*.
5. If M accepts, *reject*. If M rejects, *accept*."

a) Show that if we change Step 4 to simply "Simulate M on input w", then D is not a decider anymore.

b) Show that if we change "$f(n)$" to "$\lceil f(n)/2 \rceil$" in Step 2, and "$2^{f(n)}$" to "$2^{\lceil f(n)/2 \rceil}$" in Step 4, then $L(D)$ still cannot be decided in space $o(f(n))$.
Hint: show that for all M using space $o(f(n))$, D *accepts* $\langle M \rangle 10^\ell$ if and only if M *rejects* $\langle M \rangle 10^\ell$ for some sufficiently large $\ell \in \mathbb{N}$.