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Complexity Theory

SS 2016

Homework 4

Please submit your solutions via email to Gennadij or put it into the box at F2.111.

Exercise 1 (4 points)**:**
Let $A, B \subseteq \{0,1\}^*$ be languages and $\mathbf{C}_1, \mathbf{C}_2 \subseteq \mathcal{P}(\{0,1\}^*)$ complexity classes. Show that

a) if $A \subseteq B$, then $\bar{B} \subseteq \bar{A}$.

b) if $\mathbf{C}_1 \subseteq \mathbf{C}_2$, then co-$\mathbf{C}_1 \subseteq$ co-$\mathbf{C}_2$.

c) if co-$\mathbf{C}_1 \subseteq \mathbf{C}_1$, then $\mathbf{C}_1 =$ co-$\mathbf{C}_1$.

d) $\mathbf{P} \subseteq \mathbf{NP} \cap$ co-$\mathbf{NP}$.

Exercise 2 (4 points)**:**
Let $s : \mathbb{N} \to \mathbb{N}$ be space-constructible. Show that

a) $\mathbf{NSPACE}(s(n)) \subseteq$ co-$\mathbf{NSPACE}(s(n)^2)$.

b) co-$\mathbf{NSPACE}(s(n)) \subseteq \mathbf{NSPACE}(s(n)^2)$.

c) $\mathbf{NPSPACE} =$ co-$\mathbf{NPSPACE}$.

Exercise 3 (8 points)**:**
Let $G = (V, E)$ be an (undirected) graph. A *vertex cover* in G is a set $U \subseteq V$ such that the nodes in U cover all edges, i.e. for any $(u, v) \in E$, $u \in U$ or $v \in U$. Let

$$\textit{VertexCover} := \{\langle G, k\rangle \mid G \text{ is a graph that contains a vertex cover of size at most } k\}$$

- Show that *3SAT* $\leq_P$ *VertexCover*.

Hint: First consider the following two facts:

a) Any vertex cover of K_2 (Figure 1) requires at least one node.

b) Any vertex cover of K_3 (Figure 2) requires at least two nodes.

Figure 1: The Graph K_2

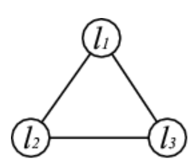

Figure 2: The Graph K_3

Find a representation of a Boolean formula ϕ as a graph G_ϕ. For this, the graph G_ϕ should contain two types of subgraphs ("gadgets"): ones that represent the assignment of 0 or 1 to a variable, and ones that represent the clauses of ϕ.
You need to ensure that ϕ is satisfiable if and only if G_ϕ has a vertex cover of a certain size.