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Complexity Theory

SS 2016

Class Handout 11

Exercise 1:
Describe the error in the following fallacious "proof" that $\mathbf{P} \neq \mathbf{NP}$:
Proof by contradiction: Assume that $\mathbf{P} = \mathbf{NP}$*. Then* $\mathrm{SAT} \in \mathbf{P}$ *and hence,* $\mathrm{SAT} \in \mathbf{DTIME}\left(n^k\right)$ *for some* k*. Since every* $L \in \mathbf{NP}$ *is poly time reducible to* SAT*, we have* $\mathbf{NP} \subseteq \mathbf{DTIME}\left(n^k\right)$*. Therefore,* $\mathbf{P} \subseteq \mathbf{DTIME}\left(n^k\right)$*. But, by the time hierarchy theorem,* $\mathbf{DTIME}\left(n^{k+1}\right)$ *contains a language that is not in* $\mathbf{DTIME}\left(n^k\right)$*, which contradicts* $\mathbf{P} \subseteq \mathbf{DTIME}\left(n^k\right)$*. We deduce that* $\mathbf{P} \neq \mathbf{NP}$*.*

Exercise 2:
Define the unique-SAT problem to be

$$\mathrm{USAT} = \{\langle\phi\rangle \mid \phi \text{ is a Boolean formula that has a single satisfying assignment}\} \quad .$$

Show that $\mathrm{USAT} \in \mathbf{P}^{\mathrm{SAT}}$.

Exercise 3:
Refute:

a) For every OTM $M^?$ and all $A, B \subseteq \Sigma^*$ it holds

$$L\left(M^A\right) = L\left(M^B\right) \quad .$$

b) For every OTM $M^?$ and all $A, B \subseteq \Sigma^*$ it holds

$$L\left(M^A\right) \neq L\left(M^B\right) \quad .$$

Exercise 4:
Show that: Savitch's theorem holds relatively to every oracle. That is, for every oracle $A \subseteq \Sigma^*$ and every space constructible function $s(n) \geq n$ it holds

$$\mathbf{NSPACE}\big(s\left(n\right)\big)^A \subseteq \mathbf{DSPACE}\left(s\left(n\right)^2\right)^A \quad .$$