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Complexity Theory

SS 2016

Class Handout 14

Exercise 1:
Using the result of Theorem 8.7 ($\mathbf{BPP} \subseteq \Sigma_2$), prove the Corollary 8.8, that is

$$\mathbf{BPP} \subseteq \Sigma_2 \cap \Pi_2.$$

Exercise 2:
Explain why

a) $\mathbf{RP} \subseteq \mathbf{NP}$

b) $\mathbf{P} \subseteq \mathbf{ZPP} \subseteq \mathbf{RP} \subseteq \mathbf{BPP}$

Exercise 3:
For $A \subseteq \{0,1\}^m$ and $t \in \{0,1\}^m$, let $t \oplus A := \{t \oplus z \mid z \in A\}$ as in the proof for $\mathbf{BPP} \subseteq \Sigma_2$. Let $t_1, \ldots, t_n \in \{0,1\}^m$. Explain why

a) $|A| = |t \oplus A|$.

b) $t_1 \in t_2 \oplus A \Leftrightarrow t_2 \in t_1 \oplus A$.

c) $|\bigcup_{i=1}^{n} t_i \oplus A| \leq n \cdot |A|$

Exercise 4:
Give an inclusion diagram of the complexity classes introduced in the lecture:

$$\mathbf{P},\ \mathbf{NP},\ \text{co-}\mathbf{NP},\ \mathbf{NPC},\ \mathbf{PSPACE},\ \mathbf{NPSPACE},$$
$$\mathbf{L},\ \mathbf{NL},\ \text{co-}\mathbf{NL},$$
$$\mathbf{NP}^{TQBF},\ \mathbf{P}^{TQBF},$$
$$\mathbf{PH}, \Delta_1,\ \Delta_2,\ \Pi_1,\ \Pi_2,\ \Sigma_1,\ \Sigma_2,\ \Sigma_2 \cap \Pi_2,\ \Sigma_2 \cup \Pi_2$$
$$\mathbf{RP},\ \text{co-}\mathbf{RP},\ \mathbf{ZPP},\ \mathbf{BPP}$$

Mark known equalities and inequalities. What relations are conjectured?