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21. April 2016

Complexity Theory

SS 2016

Class Handout 2

Exercise 1:
Show that **PSPACE** is closed under the operations union, complementation, and star.

Exercise 2:
Let $M = (Q, \Sigma, \delta, q_0, F)$ be a NFA, where $\Sigma = \{0, 1\}$. Assume that there exist $w \in \{0,1\}^*$ such that $w \notin L(M)$. Argue that then there exist $w^* \in \{0,1\}^*$, $w^* \notin L(M)$ such that $|w| < 2^q$, where $q = |Q|$ is the number of states of M.
How did we use this observation in the construction of the NTM for the language $\overline{ALL_{NFA}}$?

Exercise 3:
Let

$$\phi = Q_1 x_1 Q_2 x_2 \cdots Q_k x_k \;\; \psi(x_1, \ldots, x_k)$$

be a fully quantified Boolean formula in prenex normal form. Every Q_i represents either a $\forall$ or an $\exists$ quantifier. We associate a game with ϕ as follows. Two players, called Player $\mathcal{A}$ and Player $\mathcal{E}$, take turns selecting the values of the variables $x_1, \ldots, x_k$. Player $\mathcal{A}$ selects values for the variables that are bound to $\forall$ quantifiers, whereas Player $\mathcal{E}$ selects values for the variables that are bound to $\exists$ quantifiers. The order of play is the same as that of the quantifiers at the beginning of the formula. At the end of the game we declare that Player $\mathcal{E}$ has won the game if $\psi(v_1, \ldots, v_k) = 1$, where $v_1, \ldots, v_k$ are the values that the players have selected for the variables $x_1, \ldots, x_k$, respectively. If $\psi(v_1, \ldots, v_k) = 0$ we declare that Player $\mathcal{A}$ has won.

Show that the following language is in **PSPACE**:

$$\text{FORMULA-GAME} = \left\{ \langle\phi\rangle \;\middle|\; \begin{array}{c} \text{Player } \mathcal{E} \text{ has a winning strategy in the} \\ \text{formula game associated with } \phi. \end{array} \right\} .$$