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Complexity Theory

SS 2016

Class Handout 9

Exercise 1:
Recall that a directed graph is strongly connected if every two nodes are connected by a directed path in each direction. Show that

$$\text{STRONGLY-CONNECTED} = \{\langle G\rangle \mid G \text{ is a directed, strongly connected graph}\}$$

is a **NL**-complete language.

Exercise 2:
Let

$$\text{3Clique} = \left\{ \langle G\rangle \;\middle|\; \begin{array}{c} G=(V,E) \text{ is an undirected graph and there exist three nodes} \\ u,v,w \in V \text{ such that } \{u,v\},\{u,w\},\{v,w\} \in E. \end{array} \right\},$$

$$\text{k-Clique} = \left\{ \langle k, G\rangle \;\middle|\; \begin{array}{c} k \in \mathbb{N} \text{ and } G=(V,E) \text{ is an undirected graph and} \\ \text{there exist a } k \text{ clique in } G. \end{array} \right\}.$$

a) Show that 3Clique $\in \mathbf{L}$.

b) Is k-Clique $\in \mathbf{L}$?

Exercise 3:
One can show that the reduction function in the Cook-Levin Theorem is a log space computable function.

a) How important is this observation for the classes inside **NP**?

b) In particular, what does this observation mean for the classes **L** and **NL**?

Exercise 4:
Let

$$\text{2UnSAT} = \{\langle \phi\rangle \mid \phi \text{ is an unsatisfiable Boolean formula in 2CNF}\} .$$

Show that PATH $\leq_L$ 2UnSAT.

Hint: For a reduction of PATH to 2UnSAT consider clauses of the form $(x \to y)$.