V. Practical constructions of PRPs

- consider constructions widely used in practice
- very efficient, but security only heuristic
- consider theoretical constructions satisfying definition of pseudorandom permutations in next chapter
- see most important design principles and techniques for practical constructions
- Feistel ciphers and DES
- SPNs and AES (high level)

Truly random permutations

$$\mathrm{Perm}_n := \left\{ f : \{0,1\}^n \to \{0,1\}^n \;\middle|\; f \text{ is a permutation} \right\}$$

$$\left|\mathrm{Perm}_n\right| = 2^n!$$

random permutation: $f \leftarrow \mathrm{Perm}_n$

Pseudorandom permutation (PRP)

Definition 3.5 Let $F : \{0,1\}^* \times \{0,1\}^* \to \{0,1\}^*$ be a keyed, efficient and length-preserving permutation. F is called a pseudorandom permutation, if for all ppt distinguishers D there is a negligible function μ such that for all $n \in \mathbb{N}$

$$\left| \Pr\left[D^{F_k(\cdot)}\left(1^n\right) = 1 \right] - \Pr\left[D^{f(\cdot)}\left(1^n\right) = 1 \right] \right| \leq \mu(n),$$

where $k \leftarrow \{0,1\}^n$, $f \leftarrow \mathrm{Perm}_n$.

Block ciphers

Definition 5.1 A block cipher is an efficient, keyed permutation

$F : \{0,1\}^l \times \{0,1\}^n \to \{0,1\}^n$.

n is called the block length, l is called the key length.

permutation $\forall k \in \{0,1\}^l$

$$F_k : \{0,1\}^n \to \{0,1\}^n$$

$$x \mapsto F(k,x) = F_k(x)$$

is a bijection

efficient there is a practical algorithm A that on input

$k \in \{0,1\}^l$ and $x \in \{0,1\}^n$ outputs $F_k(x)$.

Use and security of block ciphers

- block ciphers used as building blocks for encryption schemes not as encryption schemes
- security only heuristic, but based on principles
- heuristic security: best known techniques cannot break block cipher
- principles: „algorithms will be judged on the following factors:
  - the extent to which the algorithm output is indistinguishable from a random permutation on the input block.“

(NIST on AES selection criteria)

Attacks on block ciphers

ciphertext-only attack — attacker only given series of outputs $\{F_k(x_i)\}$ for unknown $\{x_i\}$

known-plaintext attack — attacker given pairs of input/output $\{(F_k(x_i), x_i)\}$

chosen-plaintext attack — attacker given pairs of input/output $\{(F_k(x_i), x_i)\}$ for $\{x_i\}$ of his choice

chosen-ciphertext attack — attacker given pairs $\{(F_k(x_i), x_i)\}$ $\{(F_k^{-1}(y_i), y_i)\}$ for $\{x_i\}, \{y_i\}$ of his choice

Goals of adversaries

- key recovery
- distinguish from (pseudo-) random permutation

Confusion and diffusion

Confusion The relation between key/input and output should be complex, i.e. even small changes in key/input should lead to unpredictable changes in output.

Diffusion Small changes in key/input should lead to many changes in output.

Confusion and diffusion help to protect against

- linear cryptanalysis
- differential cryptanalysis.

Design of block ciphers

Design principles

- simple operations (efficiency)
- at least one non-linear operation (security)
- several rounds (security and efficiency) that result in confusion and diffusion
- 2 important structures
  - Feistel-ciphers (DES)
  - substitution-permutation networks (AES).

Feistel ciphers

Feistel ciphers defined by

1. input space $P = \{0,1\}^n$, output space $C = \{0,1\}^n$, key space $K \subseteq \{0,1\}^m$, where n is even, n=2t.

2. number of rounds r > 1.

3. KeySchedule: $\{0,1\}^m \rightarrow \{0,1\}^{r \cdot l}$

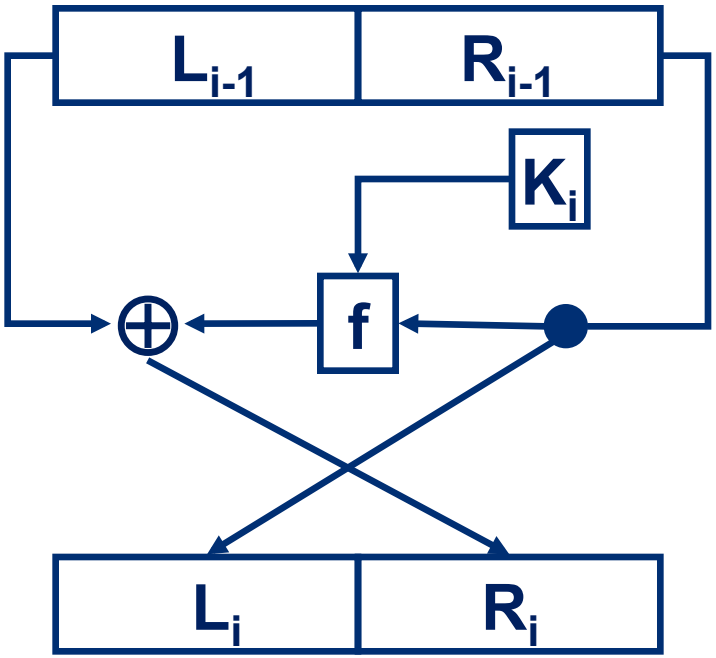
   $K \quad \mapsto K_1,\ldots,K_r$

4. round function $f : \{0,1\}^l \times \{0,1\}^t \rightarrow \{0,1\}^t$

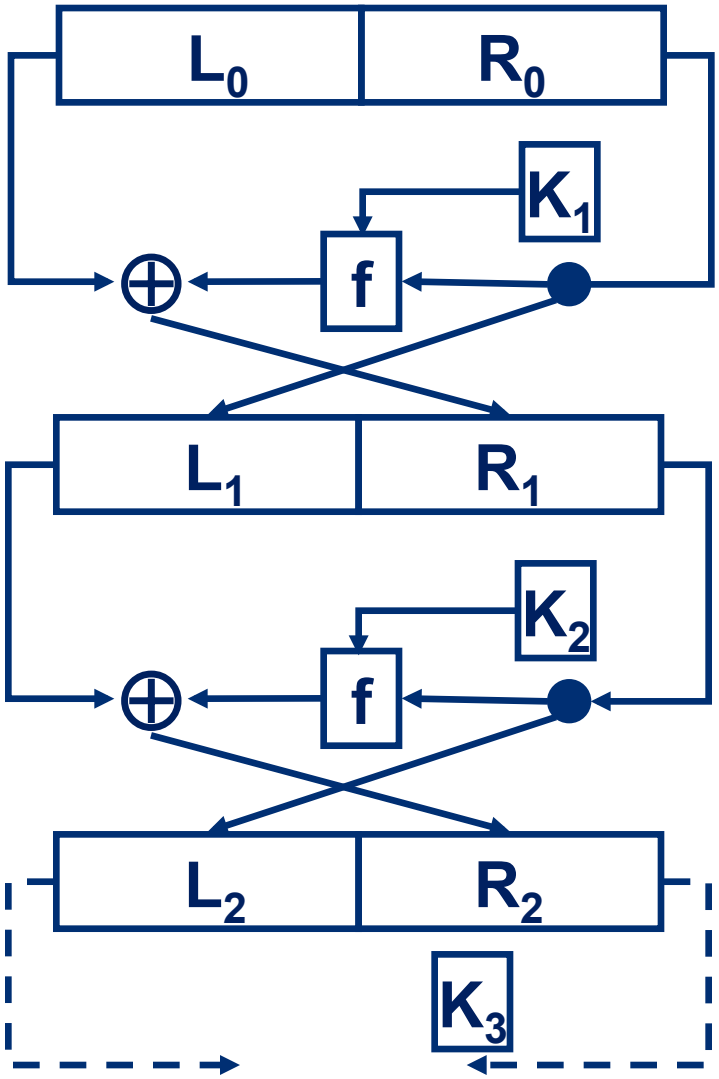
Single round

$(L_{i-1}, R_{i-1}) \mapsto (L_i, R_i)$, where

- $L_{i-1}, L_i, R_{i-1}, R_i \in \{0,1\}^t$
- $L_i = R_{i-1}, R_i = L_{i-1} \oplus f(K_i, R_{i-1})$

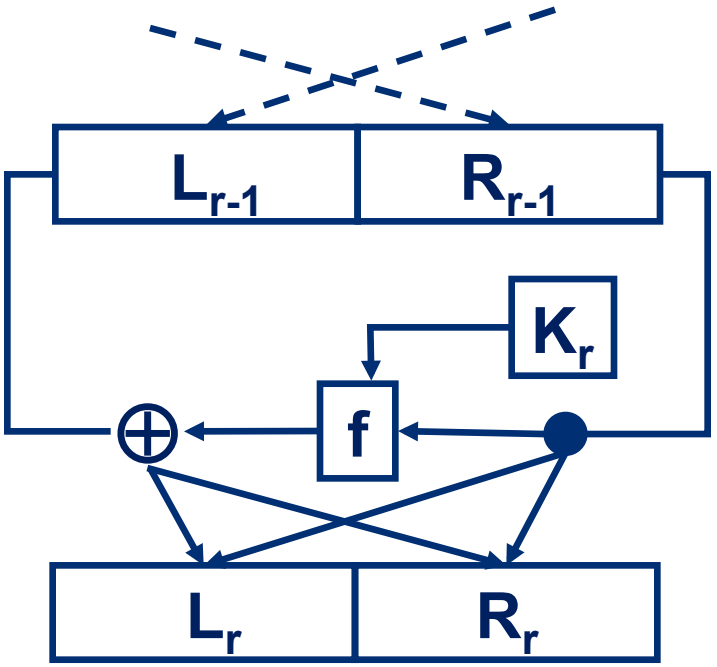


Overall structure



input $p=(L_0,R_0)$

output $c=(R_r,L_r)$



DES

- DES designed 1973/74 by NSA and IBM
- 1975 introduced as standard of NIST
- 1999 DeepCrack breaks DES in 22 hours
- DES replaced by 3DES as standard
- 2002 DES replaced by AES (Advanced Encryption Standard) as standard of NIST
- 3DES still in use today

DES setup

DES is a Feistel cipher

1. input space $P = \{0,1\}^{64}$, output space $C = \{0,1\}^{64}$, key space $K \subseteq \{0,1\}^{64}$.

2. number of rounds r =16.

3. KeySchedule: $\{0,1\}^{64} \rightarrow \{0,1\}^{16 \cdot 48}$

   $K \mapsto K_1, \ldots, K_{16}$

4. round function $f : \{0,1\}^{48} \times \{0,1\}^{32} \rightarrow \{0,1\}^{32}$

DES key space

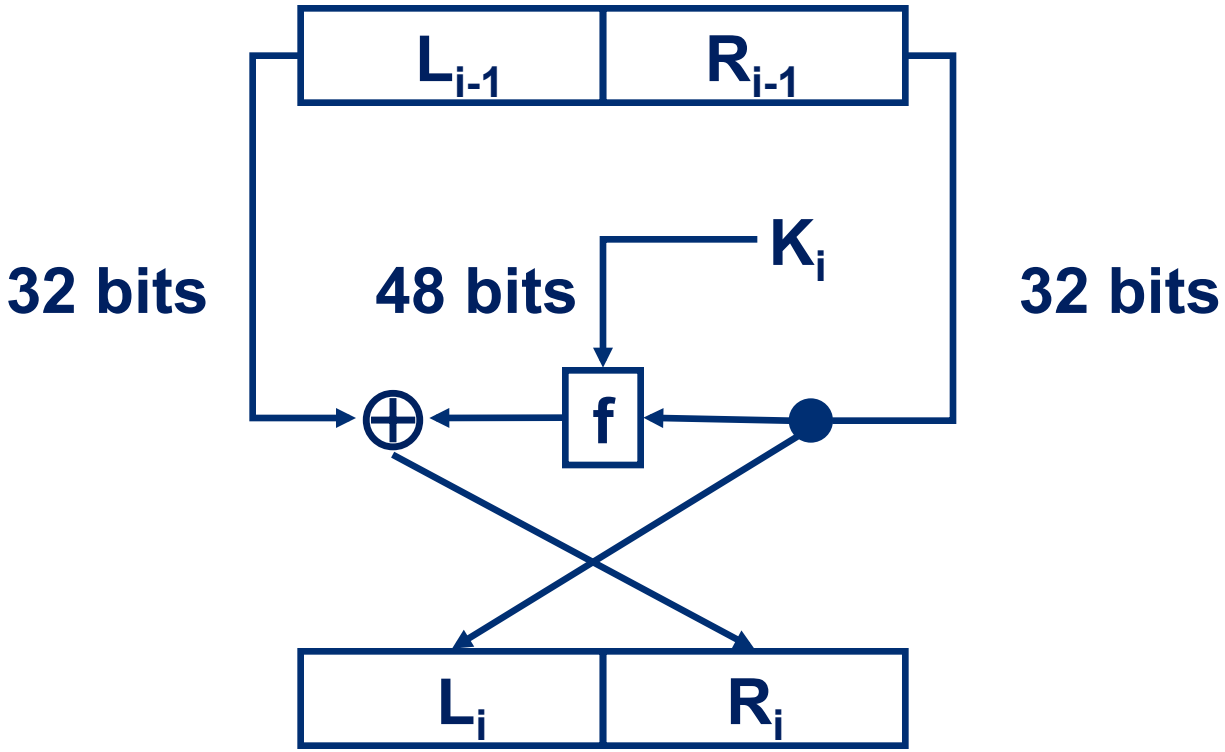
$$K = \left\{ (b_1, \ldots, b_{64}) \in \{0,1\}^{64} : \sum_{i=1}^{8} b_{8u+i} = 1 \bmod 2, 0 \le u \le 7 \right\}$$

Hence $|K| = 2^{56}$.

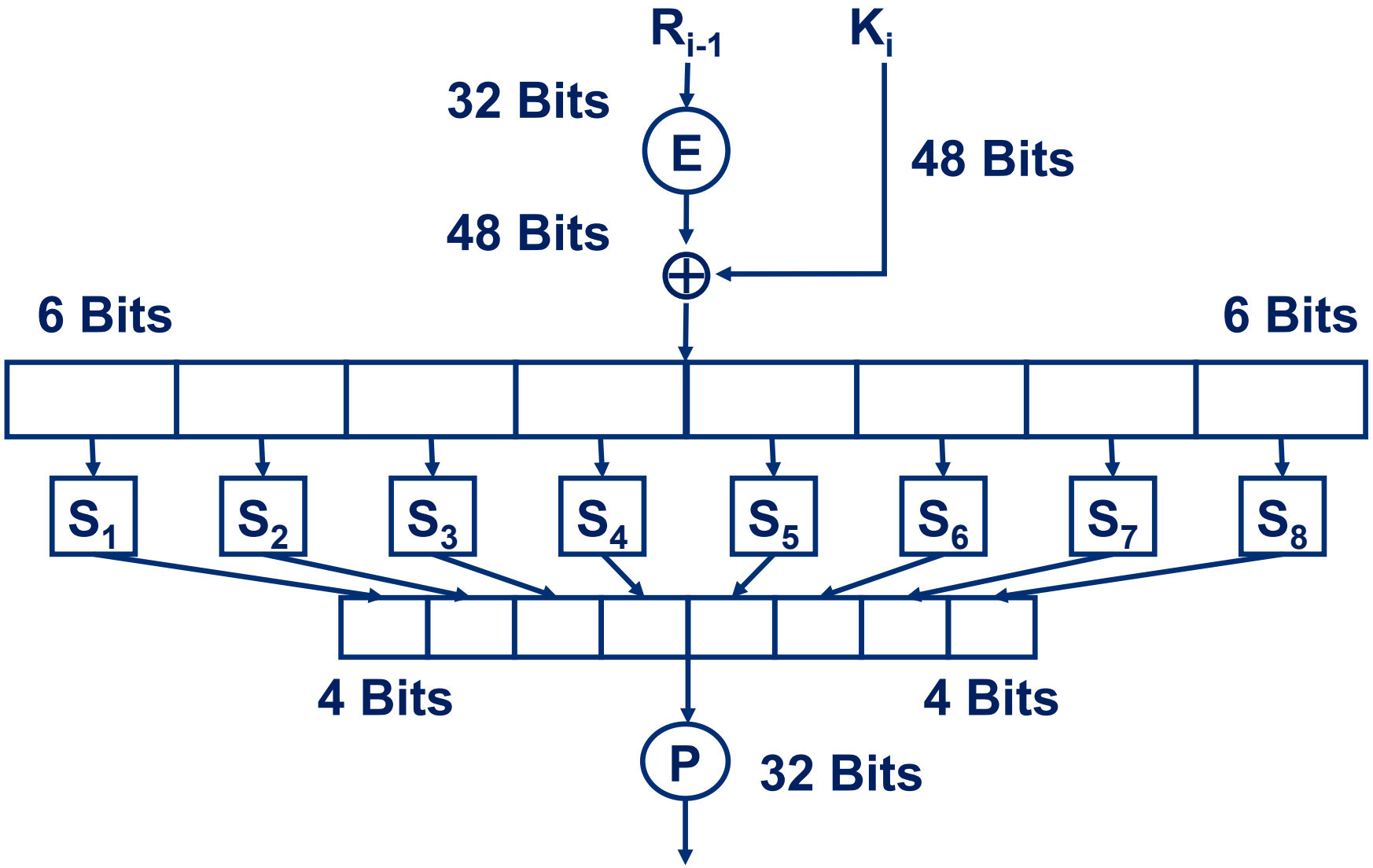
valid DES key

0	0	0	1	0	0	1	1
0	0	1	1	0	1	0	0
0	1	0	1	0	1	1	1
0	1	1	1	1	0	0	1
1	0	0	1	1	0	1	1
1	0	1	1	1	1	0	0
1	1	0	1	1	1	1	1
1	1	1	1	0	0	0	1

Single round



Round function



$$f(K_i,R_{i-1}) = P(S(E(R_{i-1}) \oplus K_i))$$

Expansion and permutation

expansion

$$E: \{0,1\}^{32} \rightarrow \{0,1\}^{48}$$

$$R_1 \cdots R_{32} \mapsto R_{32}R_1 \cdots R_{32}R_1$$

32	1	2	3	4	5
4	5	6	7	8	9
8	9	10	11	12	13
12	13	14	15	16	17
16	17	18	19	20	21
20	21	22	23	24	25
24	25	26	27	28	29
28	29	30	31	32	1

permutation

$$P: \{0,1\}^{32} \rightarrow \{0,1\}^{32}$$

$$R_1 \cdots R_{32} \mapsto R_{16} \ldots R_{25}$$

16	7	20	21
29	12	28	17
1	15	23	26
5	18	31	10
2	8	24	14
32	27	3	9
19	13	30	6
22	11	4	25

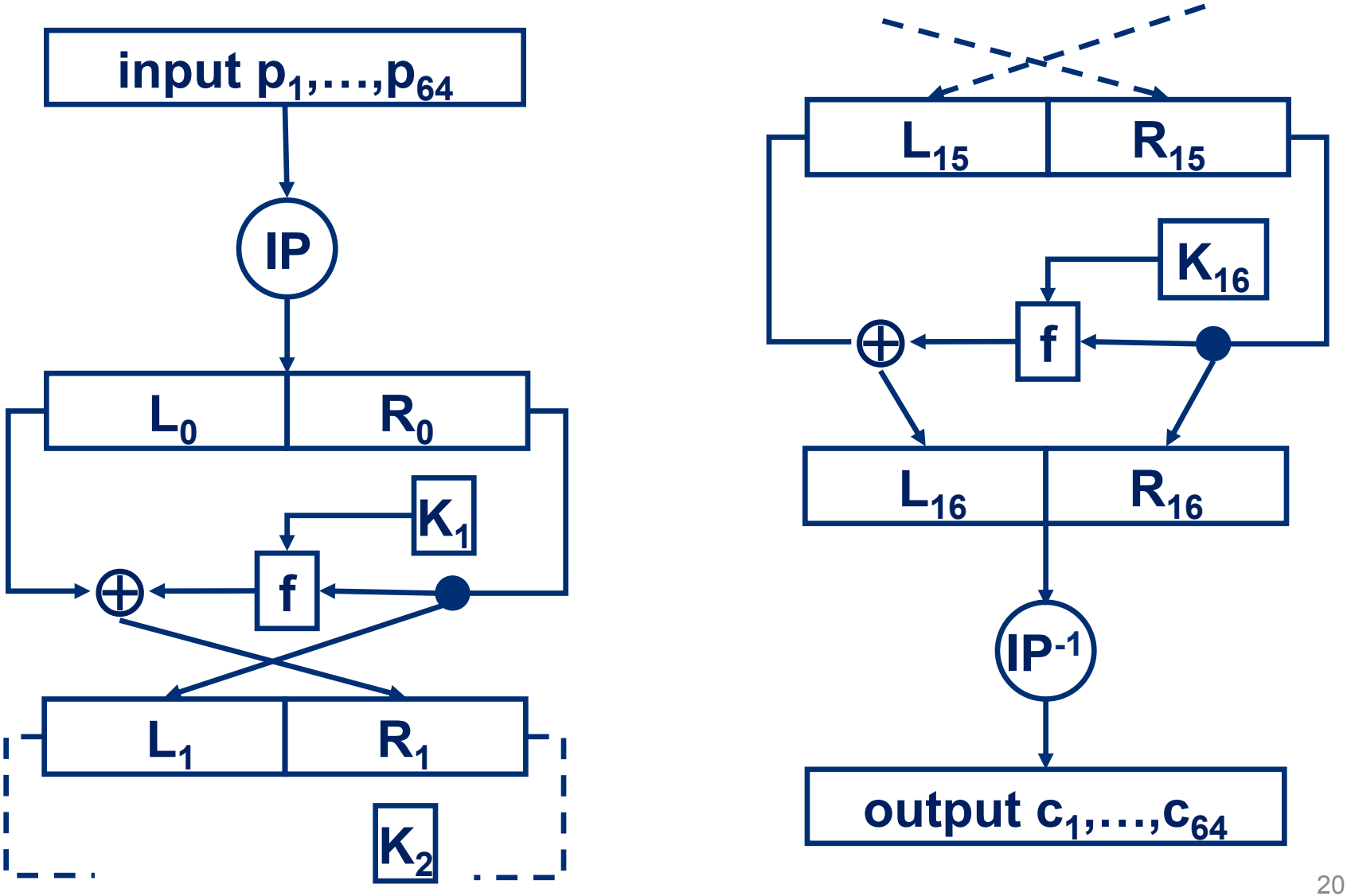
S-boxes

$$S_i : \{0,1\}^6 \rightarrow \{0,1\}^4 \qquad ,i = 1,\ldots,8$$
$$b_1 \ldots b_6 \mapsto S_i(b_1 b_6, b_2 b_3 b_4 b_5)$$

1. interpret $b_1 b_6$ as row index, $b_2 b_3 b_4 b_5$ as column index.
2. output is binary representation of entry in position $(b_1 b_6, b_2 b_3 b_4 b_5)$ of i-th S-Box.

S_1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	14	4	13	1	2	15	11	8	3	10	6	12	5	9	0	7
1	0	15	7	4	14	2	13	1	10	6	12	11	9	5	3	8
2	4	1	14	8	13	6	2	11	15	12	9	7	3	10	5	0
3	15	12	8	2	4	9	1	7	5	11	3	14	10	0	6	13

DES


input $p_1,\ldots,p_{64}$
IP
L_0
R_0
K_1
f
L_1
R_1
K_2
L_{15}
R_{15}
K_{16}
f
L_{16}
R_{16}
IP^{-1}
output $c_1,\ldots,c_{64}$

Initial permutation

$$IP: \{0,1\}^{64} \rightarrow \{0,1\}^{64}$$

$$p_1 \cdots p_{64} \mapsto p_{58} p_{50} \cdots p_7$$

$$IP^{-1}: \{0,1\}^{64} \rightarrow \{0,1\}^{64}$$

$$v_1 \cdots v_{64} \mapsto v_{40} v_8 \cdots v_{25}$$

58	50	42	34	26	18	10	2
60	52	44	36	28	20	12	4
62	54	46	38	30	22	14	6
64	56	48	40	32	24	16	8
57	49	41	33	25	17	9	1
59	51	43	35	27	19	11	3
61	53	45	37	29	21	13	5
63	55	47	39	31	23	15	7

40	8	48	16	56	24	64	32
39	7	47	15	55	23	63	31
38	6	46	14	54	22	62	30
37	5	45	13	53	21	61	29
36	4	44	12	52	20	60	28
35	3	43	11	51	19	59	27
34	2	42	10	50	18	58	26
33	1	41	9	49	17	57	25

S-boxes

Lemma 5.2 For $i = 1,\ldots,8$ and for all $u,v \in \{0,1\}^6$, that differ in exactly one position, $S_i(u)$ and $S_i(v)$ differ in at least two positions.

Example $i = 1, u = 101101, v = 111101$

$$S_1(u) = 0001, S_1(v) = 0110$$

S_1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	14	4	13	1	2	15	11	8	3	10	6	12	5	9	0	7
1	0	15	7	4	14	2	13	1	10	6	12	11	9	5	3	8
2	4	1	14	8	13	6	2	11	15	12	9	7	3	10	5	0
3	15	12	8	2	4	9	1	7	5	11	3	14	10	0	6	13

Permutation

Lemma 5.3 View $u,v \in \{0,1\}^{32}$ as sequence of eight 4-bit sequences. If u,v differ in two positions in a single 4-bit string, then $P(u)$ and $P(v)$ differ in two 4-bit sequences.

Example $u = 0^{32}, v = 110^{30}$

$P(u) = 0^{32}$

$P(v) = 0^8 10^7 10^{15}$

16	7	20	21
29	12	28	17
1	15	23	26
5	18	31	10
2	8	24	14
32	27	3	9
19	13	30	6
22	11	4	25

Diffusion in DES

Lemma 5.2 For $i = 1,\ldots,8$ and for all $u,v \in \{0,1\}^6$, that differ in exactly one position, $S_i(u)$ and $S_i(v)$ differ in at least two positions.

Lemma 5.3 View $u,v \in \{0,1\}^{32}$ as sequence of eight 4-bit sequences. If u,v differ in two positions in a single 4-bit string, then $P(u)$ and $P(v)$ differ in two 4-bit sequences.

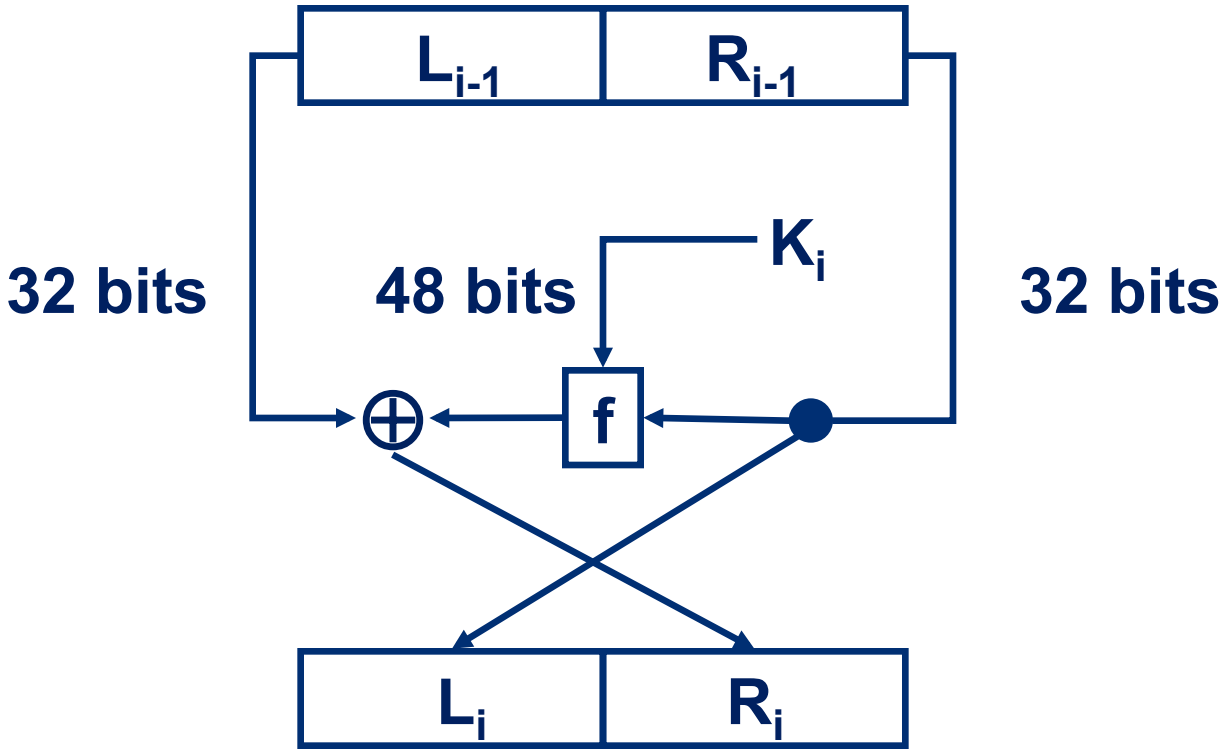
Diffusion and avalanche effect

inputs p and q differ in Δ=1 positions, difference in left half

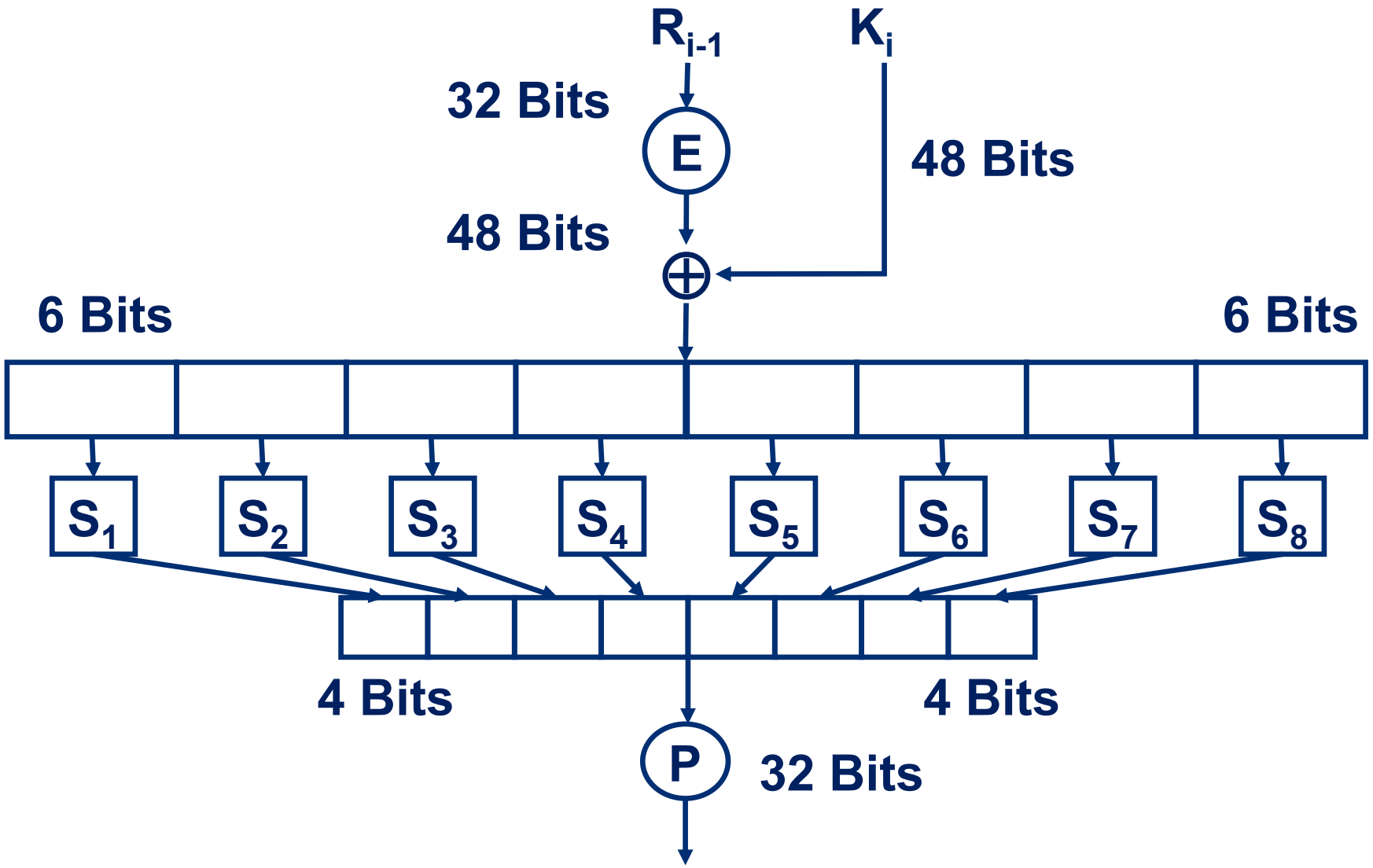
Δ=1	Δ=0	input
Δ=0	Δ=1	after round 1
Δ=1	Δ=2	after round 2
Δ=2	Δ=4	after round 3
Δ=4	Δ=8	after round 4

avalanche effect

Single round



Round function



$$f(K_i,R_{i-1}) = P(S(E(R_{i-1}) \oplus K_i))$$

DES variants

2DES $P = C = \{0,1\}^{64}, K \subseteq \{0,1\}^{64} \times \{0,1\}^{64}$

$k \in K, k = (k_1, k_2)$

$2DES_k(p) = DES_{k_2}(DES_{k_1}(p))$

3DES $P = C = \{0,1\}^{64}, K \subseteq \{0,1\}^{64} \times \{0,1\}^{64} \times \{0,1\}^{64}$

$k \in K, k = (k_1, k_2, k_3)$

$3DES_k(p) = DES_{k_3}(DES^{-1}_{k_2}(DES_{k_1}(p)))$

- 2DES not as secure as key length indicates.
- 3DES still considered secure, but too slow.

Substitution-permutation networks

SPNs defined by

1. input space $P = \{0,1\}^n$, output space $C = \{0,1\}^n$, key space $K = \{0,1\}^n$, where $n = t \cdot b$.

2. round number $r > 1$.

3. KeySchedule: $\{0,1\}^n \rightarrow \{0,1\}^{(r+1)\cdot n}$

   $K \mapsto K_0, \ldots, K_r$

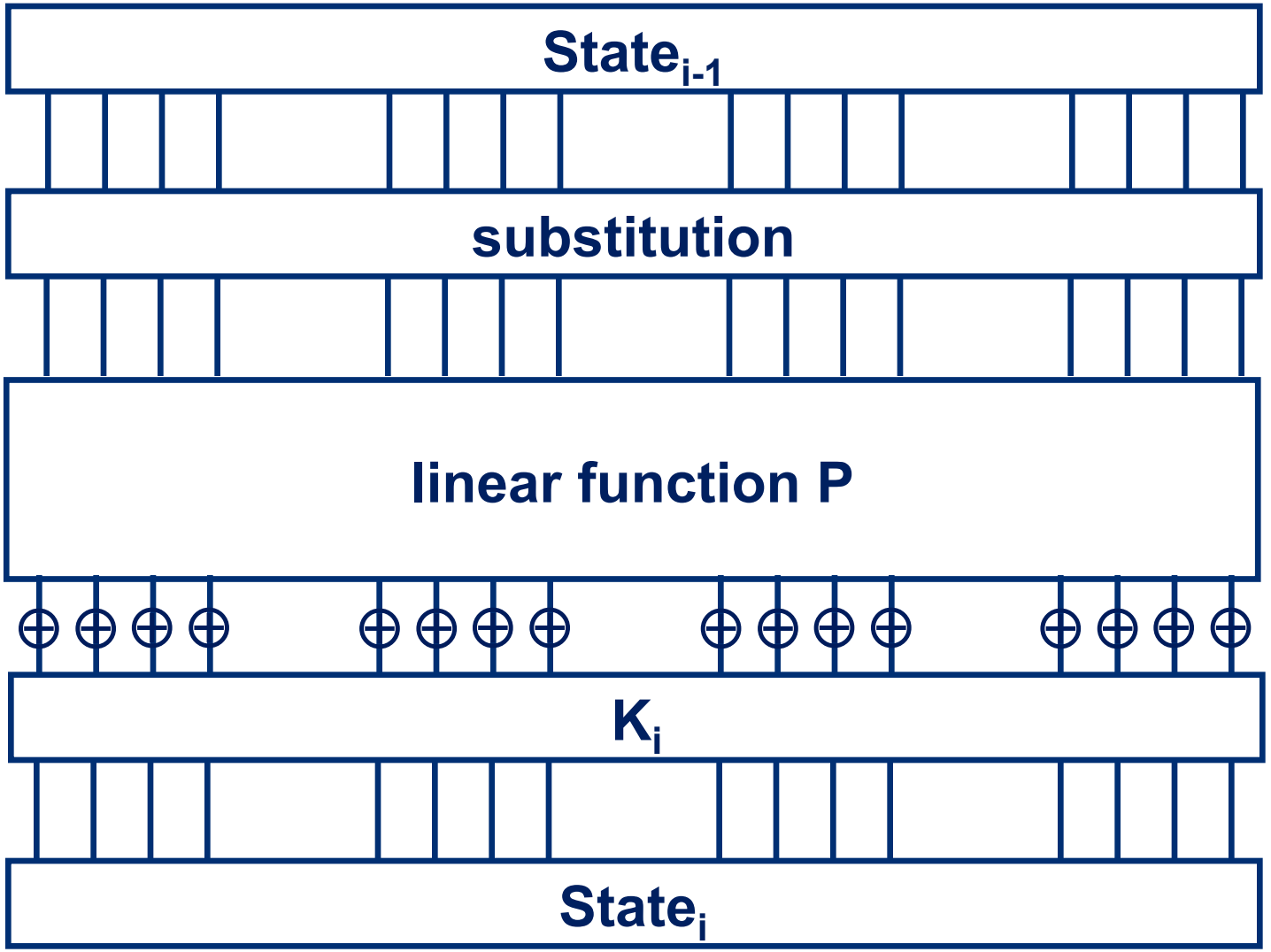
4. bijective substitutions $S_i : \{0,1\}^b \rightarrow \{0,1\}^b, i = 1, \ldots, t$

5. bijective linear function $P : \{0,1\}^n \rightarrow \{0,1\}^n$.

(P sometimes permutation on n bits, hence the name)

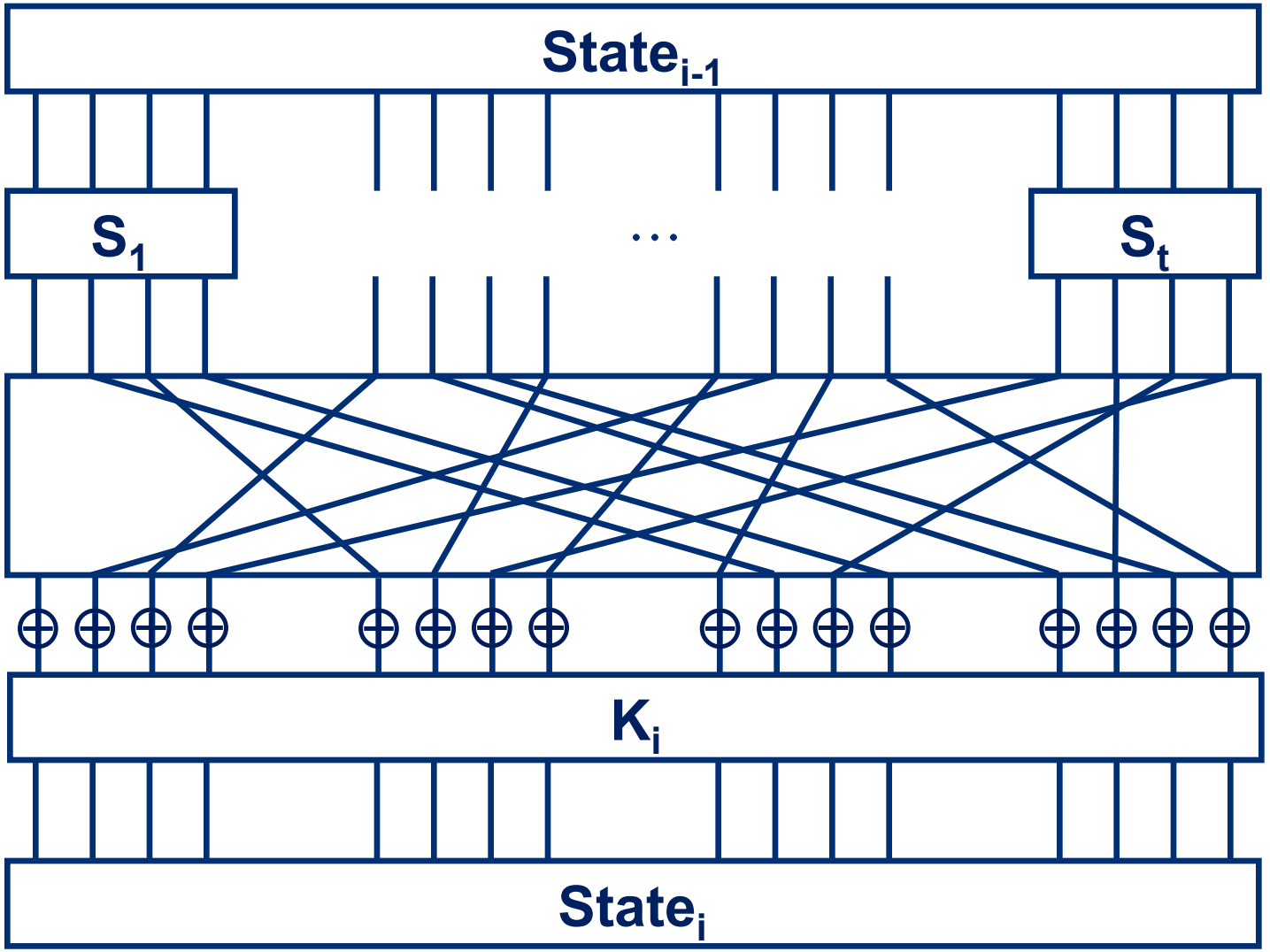
SPN – single round

$$State_i := P\left(S\left(State_{i-1}\right)\right) \oplus K_i, \quad State_j \in \{0,1\}^n$$



SPN – single round

$$State_i := P\left(S\left(State_{i-1}\right)\right) \oplus K_i, \quad State_j \in \{0,1\}^n$$



SPN initialization

- input $p \in \{0,1\}^n$
- $State_0 = p \oplus K_0$
- called whitening
- introduced so that all states depend on the secret key

Advanced Encryption Standard (AES)

- 1997 NIST call for proposals for successor of DES.
- 1998 15 proposals submitted.
- 1999 selection of 5 candidates (MARS, RC6, Rijndael, Serpent, Twofish).
- 2. Oktober 2000 announcement of Rijndael as winner.
- 26. November 2001 AES-Rijndael replaces DES as NIST standard.
- AES-Rijndael designed by Joan Daemen and Vincent Rijmen.

AES parameters

AES is SPN cipher with

1. input space $P = \{0,1\}^{128}$, output space $C = \{0,1\}^{128}$, key space $K = \{0,1\}^{128}$, where b=8,t=16.

2. round number r =10.

3. KeySchedule: $\{0,1\}^{128} \rightarrow \{0,1\}^{11\cdot 128}$

   $K \mapsto K_0,\ldots,K_{10}$

4. bijective substitution $S : \{0,1\}^8 \rightarrow \{0,1\}^8$

5. linear bijection $P : \{0,1\}^{128} \rightarrow \{0,1\}^{128}$.

States in AES

$$\text{State}_j = \begin{array}{|c|c|c|c|} \hline B_{00} & B_{01} & B_{02} & B_{03} \\ \hline B_{10} & B_{11} & B_{12} & B_{13} \\ \hline B_{20} & B_{21} & B_{22} & B_{23} \\ \hline B_{30} & B_{31} & B_{32} & B_{33} \\ \hline \end{array}$$

$$B_{ij} \in \{0,1\}^8, i, j \in \{0,1,2,3\}$$

AES round structure

State ← p

State ←AddRoundKey(State, K_0)

For i=1 to 9 do

    State← SubBytes(State)

    State← ShiftRows(State)

    State← MixColumns(State)

} function P

    State← AddRoundKey(State, K_i)

State← SubBytes(State)

State← ShiftRows(State)

State← AddRoundKey(State, K_{10})

return State

Byte interpretation

$$\{0,1\}^8 = \{0,1\}^4 \times \{0,1\}^4$$

$$\{0,1\}^4 \doteq \{0,1,2,\ldots,15\}$$

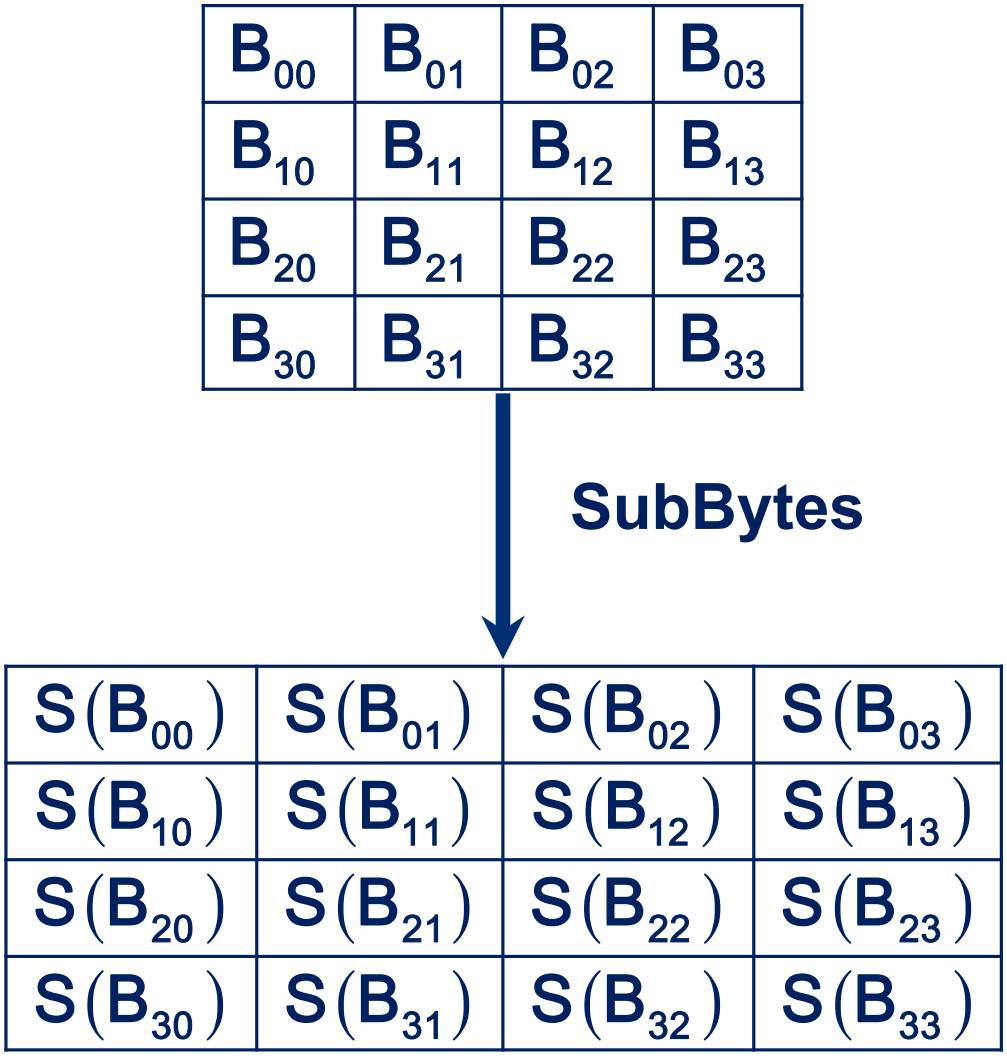
write elements in $\{0,1,2,\ldots,15\}$ in hexadecimal representation.

$$\{0,1,2,\ldots,14,15\} \doteq \{0,1,2,\ldots,9,A,B,C,D,E,F\}$$

Fact There is a field $\mathbb{F}_{256}$ with 256 elements.

$256 = 2^8 \quad \Rightarrow$ identify elements in $\mathbb{F}_{256}$ with pairs of hexadecimal numbers

SubBytes

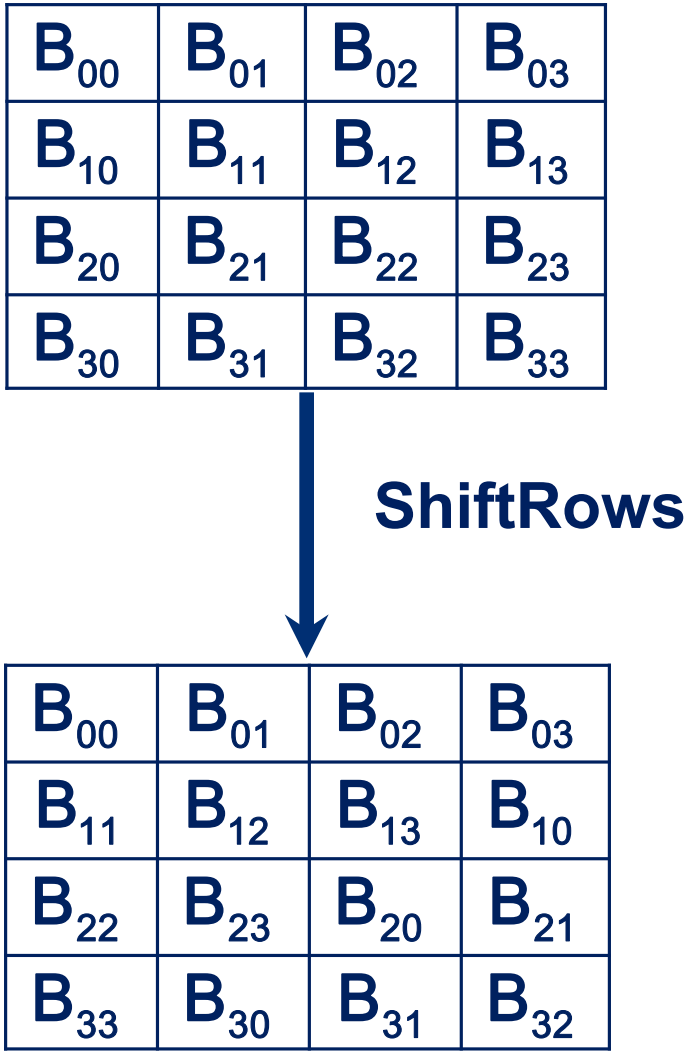


$S : \{0,1\}^8 \rightarrow \{0,1\}^8$ non-linear

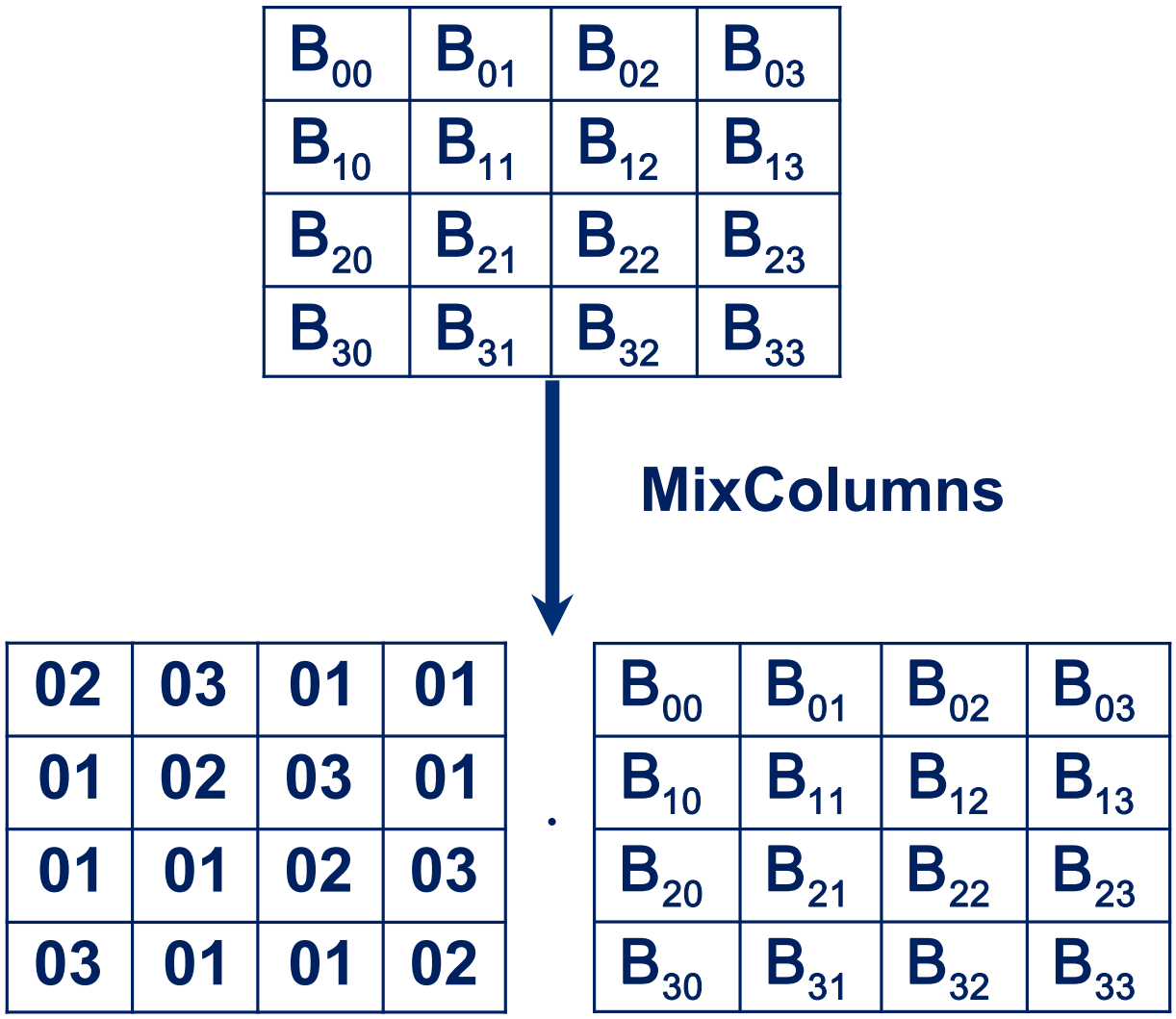
SubBytes

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
A	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
B	E7	C8	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
C	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
D	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
E	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
F	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

ShiftRows


B_{00} B_{01} B_{02} B_{03}
B_{10} B_{11} B_{12} B_{13}
B_{20} B_{21} B_{22} B_{23}
B_{30} B_{31} B_{32} B_{33}
ShiftRows
B_{00} B_{01} B_{02} B_{03}
B_{11} B_{12} B_{13} B_{10}
B_{22} B_{23} B_{20} B_{21}
B_{33} B_{30} B_{31} B_{32}

MixColumns



Arithmetic operations performed in $\mathbb{F}_{256}$!

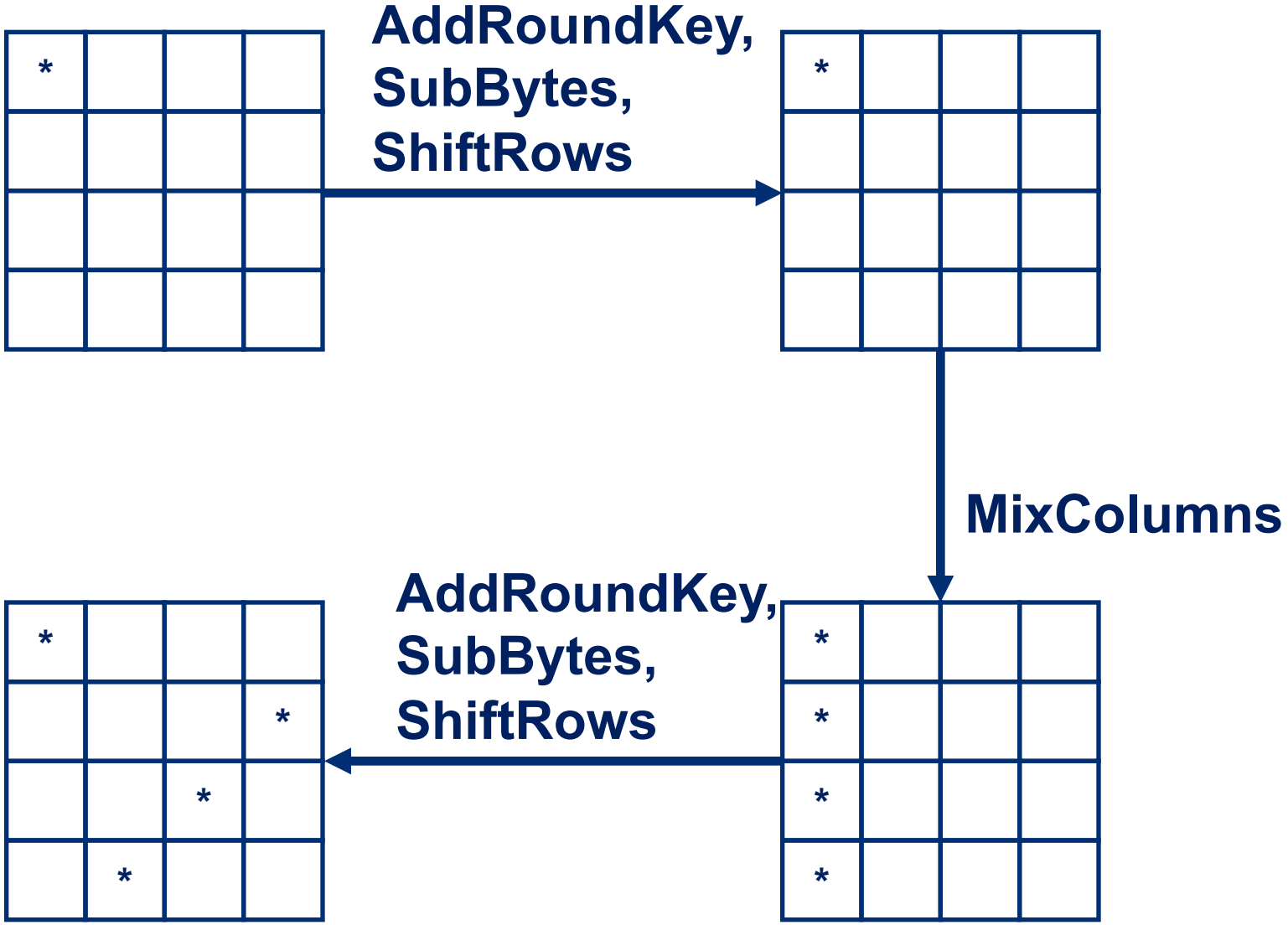
Diffusion in AES

Diffusion Small changes in key/input should lead to many changes in output.

Lemma 5.4 If a vector consisting of four bytes has t non-zero bytes, then its image under MixColumns has at least 5 – t non-zero bytes, $1 \leq t \leq 4$.

MDS property (maximum distance separable)

Diffusion in AES



* := positions with different bytes in two states

Diffusion in AES

*:= positions with different bytes in two states

Inversion in AES

State ← p

State ←AddRoundKey(State, K_{10})

For i=9 downto 1 do

    State ← InvShiftRows(State)

    State ← InvSubBytes(State)

    State ← AddRoundKey(State, K_i)

    State ← InvMixColumns(State)

State ← InvShiftRows(State)

State ← InvSubBytes(State)

State ← AddRoundKey(State, K_0)

return State

Inv*:= inverse operation to *