

## Class Exercise 4

### Exercise 1 : Cost Measure

Given an OR-graph  $G$  and the cost function

$$C_{P_{s-\gamma}}(n_k) = \sum_{i=k}^{m-1} \frac{c(n_i, n_{i+1})}{m-i}$$

where  $P_{s-\gamma} = (n_0, n_1, \dots, n_m)$ , i.e.,  $n_{i+1}$  is the successor of  $n_i$  in the solution path  $P_{s-\gamma}$  and  $P_{s-\gamma}$  has length  $m$  with  $n_0 := s$  and  $n_m := \gamma$ .

Can we give a definition of  $C_{P_{s-\gamma}}$  in form of a recursive cost function?

### Exercise 2 : Optimistic Heuristics

Given the 8-puzzle problem. Let the true cost of each state  $A$  be the minimum number of moves necessary to reach the goal state from  $s$ . Example:

1	2	3		1	2	3
4		6		4	5	6
7	8	5		7	8	
$s$			$\gamma$			

- (a) Define  $C_P(n)$  in form of a recursive cost function and define  $\widehat{C}_P(n)$  based on the definition of  $C_P(n)$  using some heuristic function  $h$ . What is the cost measure?
- (b) The heuristic cost function  $h_1$  estimates the cost of a state  $s'$  as the number of positions for which the content on that position in  $s'$  is not equal to the content on that position in the goal state. In the example above,  $h_1(s) = 2$  (1 for the central and bottom right positions each). Is  $h_1$  optimistic?
- (c) The heuristic cost function  $h_2$  estimates the cost of a state  $s'$  as the sum of the Manhattan (city block) distances between the position of each of the 8 tiles in  $s'$  and its position in the goal state. In the example above,  $h_2(s) = 2$  (tile 5 has to be moved up by 1 and left by 1). Is  $h_2$  optimistic?
- (d) Does  $h_1(s') \geq h_2(s')$  or  $h_1(s') \leq h_2(s')$  hold for all possible states  $s'$ ?

### Exercise 3 : Evaluation Function $f = g + h$ in A\*

Let  $n$  be a node in a search space graph that is explored by A\* using heuristic  $h$ .

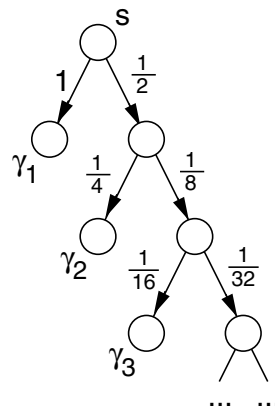
- (a) When can the value of  $h(n)$  change during A\*-search?
- (b) When can the value of  $g(n)$  change during A\*-search?

### Exercise 4 : Negative Cost Cycles

A negative-cost cycle is a cycle in a search space graph whose sum of edge weights is negative. Why are negative-cost cycles problematic for A\* search?

**Exercise 5 : Optimum Solution Paths in OR-graphs**

Given the following search space graph  $G$ :



Can you determine an optimum solution graph? If no, why not?

**Exercise 6 : State Space Search**

You need to send out all the objects listed in the following table in postal packages.

Object	A	B	C	D	E	F	G	H	I	J
Weight (kg)	1	4	6	7	7	9	11	13	15	16

One package can weigh at most 20 kilograms. The shipping costs per package are 9,90 EUR. Your goal is to pack all items into as few packages as possible.

- Represent the package-packing problem as an OR graph search. What information is represented by the nodes in the graph; what operations are represented by the edges?
- What are goal nodes in your representation, what are the characteristics of solution paths, what are dead ends?
- How do you determine the costs associated with the edges in your representation?
- Given a solution base  $P$ , can you give a lower bound on the cost of every single solution path extending  $P$ ?