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Randomized Algorithms

SS 2018

Homework Assignment 5

Problem 15:
In the p-MEDIAN problem we are given a complete graph G on n nodes with weights $c_{i,j} \in \mathbb{R}_+$ on the edges $\{i, j\}$ and the goal is to find a set $M \subseteq V$ of p *median* nodes so that the sum of the distances from each node to its closest median is minimized. The goal is first of all to formulate an ILP for this problem. For this we use the indicator variables $x_{i,j} \in \{0, 1\}$ and $y_j \in \{0, 1\}$ with the property that

- $x_{i,j} = 1$ if and only if the edge $\{i, j\}$ connects node i to the closest median node j and
- $y_j = 1$ if and only if node j is a median node.

We formulate an ILP step by step.

(a) First, state the objective function.

(b) Next, state the constraints. Note that $x_{i,j}$ should only be 1 if j is median node and only one median node should be picked by each node i. Also, exactly p median nodes should be picked.

(c) Propose a randomized rounding strategy that makes sure that we arrive at a feasible solution for the ILP. Can you analyze it?

Problem 16:
In the CYCLECOVER problem we are given a complete graph G on n nodes with weights $c_{i,j} \in \mathbb{R}_+$ on the edges $\{i, j\}$ and the goal is to find a set of edges S of minimum cost so that every node is contained in exactly two edges, i.e., S forms a set of cycles that covers all nodes.

(a) Formulate the problem as an ILP.

(b) Propose a randomized rounding strategy that makes sure that we arrive at a feasible solution for the ILP. Can you analyze it?