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Paderborn, October 13, 2017
Submission: October 20, 2017

Fundamental Algorithms

WS 2017

Exercise Sheet 1

Exercise 1:
Show the following statements:

a) $2n^3 - 4n + 2 \in \Theta(n^3)$

b) $\sqrt{n} \in o(n)$

c) $o(n) \subset O(n^2)$

d) For arbitrary functions $f : \mathbb{N} \to \mathbb{N}$ and $g : \mathbb{N} \to \mathbb{N}$ the following holds:

$$f(n) \in o(g(n)) \Leftrightarrow g(n) \in \omega(f(n))$$

Exercise 2:
Show the following statements for functions of $\mathbb{N} \to \mathbb{N}$:

a) $\forall c \in \mathbb{N} : c \cdot f(n) = O(f(n))$

b) $f(n) + g(n) = \Omega(f(n))$

c) $g(n) = O(f(n)) \Rightarrow f(n) + g(n) = O(f(n))$

d) $O(f(n)) \cdot O(g(n)) = O(f(n) \cdot g(n))$

Exercise 3:
A sequence of n operations is performed on some data structure. The i-th operation costs i, if $i = 2^k$ for some $k \in \mathbb{N}_0$, and 1, otherwise. Use the potential method to show that the cost of the sequence of operations is bounded by $O(n)$.
Hint: The states resulting from the cheap operations must accumulate enough potential such that the next expensive operation can be paid for by an appropriate drop of potential.