

## Fundamental Algorithms

### WS 2017

### Exercise Sheet 4

**Exercise 1:**

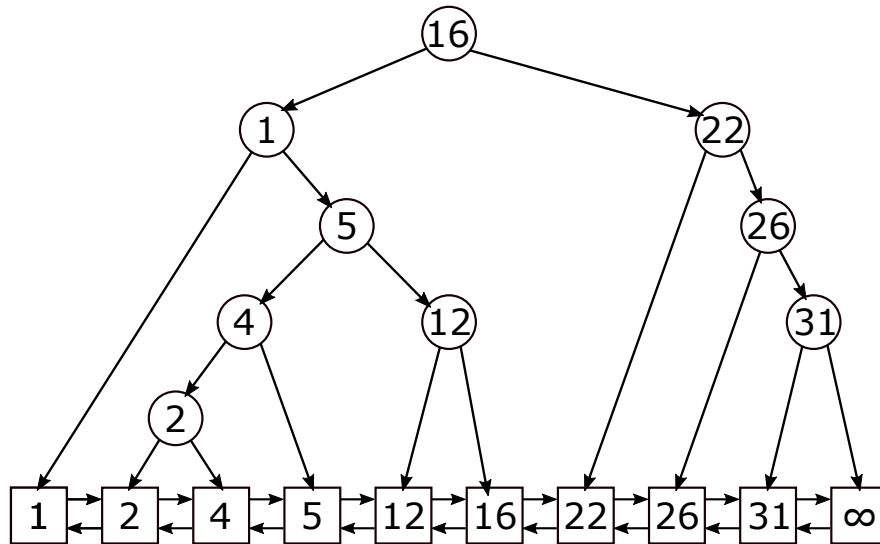
Let  $E$  be the set of all elements and superelements in an extended Radix heap and let  $\text{pos}(e)$  denote the position of  $e$  in its radix Heap. We define the following potential function on an extended Radix heap in state  $s$ .

$$\phi(s) = \sum_{e \in E} K + \text{pos}(e)$$

By using the above potential, show that in an extended Radix heap the amortized runtime of `deleteMin` and `delete` is  $O(1)$  and the amortized runtime of `insert`, `merge`, and `decreaseKey` is  $O(\log C)$ , as indicated on Slide 112.

**Exercise 2:**

Recall the Splay tree from Exercise 4 of Exercise Sheet 3:



- a) Perform the operations `search(2)`, `search(31)`, and `search(99)` (in this order) on the tree.
- b) On the result of a), perform `insert(18)` and `insert(32)` (in this order).
- c) On the result of b), perform `delete(18)` and `delete(31)` (in this order).

**Exercise 3:**

Show that any binary tree with  $n$  nodes can be converted to any other binary tree of  $n$  nodes using  $O(n)$  rotations.

*Hint:* Show that  $O(n)$  rotations suffice to convert any binary tree into a left (right) chain, where each internal node has an external right (left) child.