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Fundamental Algorithms

WS 2017

Exercise Sheet 8

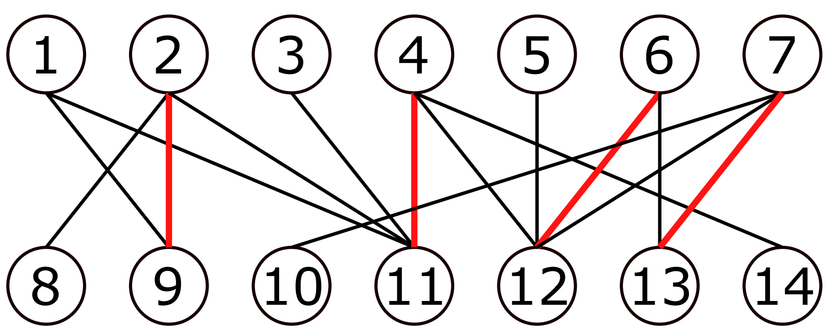
Exercise 1:
Let s be the cardinality of a maximum matching in a graph G. Show that each *maximal* matching of G has cardinality at least $\lfloor s/2 \rfloor$.

Hint: Use Lemma 5.13.

Exercise 2:
Compute a maximum matching in the below graph using the refined matching algorithm on Slide 26 of Chapter 5. In each iteration, perform the following steps:

- Firstly, compute the length of a shortest augmenting path.
- Secondly, compute a maximal set of node-disjoint shortest augmenting paths. Provide the set of paths you have found.
- Finally, give the resulting matching.

Repeat the above steps until no augmenting path can be found anymore.
Start with the given maximal matching $\{\{2,9\},\{4,11\},\{6,12\},\{7,13\}\}$.



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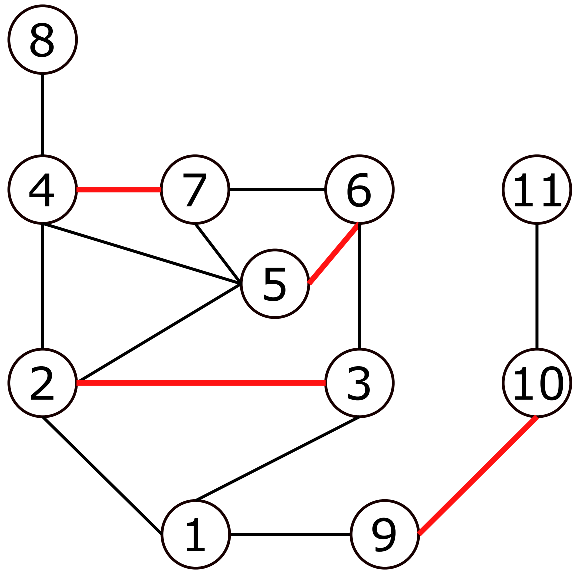
Exercise 3:
Compute of maximum matching by performing Edmonds' algorithm on the below graph. Start with the given maximal matching $\{\{2,3\},\{5,6\},\{4,7\},\{9,10\}\}$. Begin your search at node 1 and scan the neighbors of each node in ascending order of their identifier.



Exercise 4:
Assume each edge of a graph G is assigned a cost $c(e)$. We define the cost of a matching M of G to be $c(M) = \sum_{e \in M} c(e)$. A maximum matching M is a *mincost matching* if there is no maximum matching $N \neq M$ with smaller cost.
Let P be an alternating path of even length w.r.t. a matching M in which the endpoint of P that belongs to an unmatched edge in P is unmatched in M. P is a *cost-reducing path*, if $c(M) > c(M \ominus P)$. Correspondingly, define a *cost-reducing cycle* C to be an alternating cycle of even length such that $c(M) > c(M \ominus C)$.
Show the following statement by using similar arguments as in the proof of Lemma 5.12:

A maximum matching M is a mincost matching if and only if there is no cost-reducing path or cycle for M.