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Advanced Algorithms

WS 2019

Homework Assignment 2

Problem 5:
Suppose that we pick $k \leq n$ out of n chairs at some round table uniformly at random and assign them to k people. What is the expected number of people that do not have a direct table neighbor?
Hint: recall that there are $\binom{n}{k}$ ways of picking k of n chairs for the k people. Use indicator variables and the linearity of expectation to compute the expected number of people without a neighbor.

Problem 6:
Prove the second half of the Chernoff inequality, i.e., for all $0 < \delta < 1$,

$$\Pr[X \leq (1-\delta)\mu] \leq \left(\frac{e^{-\delta}}{(1-\delta)^{1-\delta}}\right)^{\mu} \leq e^{-\delta^2\mu/2}$$

Hint: Use the exponential function $g(x) = e^{-h\cdot x}$ and set $h = -\ln(1-\delta)$.

Problem 7:
Consider an arbitrary decision problem P (i.e., there are only outputs of the form "YES" or "NO"). Suppose that we have a randomized algorithm A for P with the following property:

- For all inputs $x \in P$, $\Pr[A(x) = "NO"] \leq 1/3$ and
- for all inputs $x \notin P$, $\Pr[A(x) = "YES"] \leq 1/3$.

In other words, the error probability of A is at most $1/3$. Show that by executing A $O(\log n)$ times and using an appropriate decision rule based on its outputs, one can reduce the error probability to at most $1/n$.