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Advanced Algorithms

WS 2019

Homework Assignment 4

Problem 11:
Prove Theorem 3.10.

Problem 12:
Consider the problem of distributing n balls among n bins as evenly as possible, and suppose we are using the following RANDOM strategy for this: For each ball, pick a bin uniformly and independently at random.
Compute the expected number of balls in a bin and show that with high probability (i.e., with probability at least $1 - 1/n$) every bin has at most $c \log n / \log \log n$ many balls, for a sufficiently large constant c.
(Hint: First, show that the probability is at most $1/n^2$ that some fixed bin i has at least $c \log n / \log \log n$ many balls, and conclude from that that the probability is at most $1/n$ that there is a bin with at least $c \log n / \log \log n$ many balls.)

Problem 13:
Consider the following online job scheduling problem: The input sequence σ consists of a sequence of jobs $J_1, \ldots, J_n$, where each J_i requires some time $t_i \in \mathbb{N}$ to be processed. The task of the online algorithm is to assign each job to one of m machines, $M_1, \ldots, M_m$, so that the makespan is minimized, where the makespan is the maximum over all machines of the total time needed by a machine to process the jobs assigned to it.
Suppose we use the simple online algorithm RANDOM: for each job J_i, choose a machine uniformly and independently at random.

(a) Determine the expected total time needed by a specific machine M_i to process the jobs assigned to it.

(b) Determine the variance of the total time needed by a specific machine M_i to process the jobs assigned to it.

(c) Find an upper bound on the expected makespan of algorithm RANDOM (recall that the makespan is the *maximum* total runtime assigned to a machine). For that, use either Chebychev's inequality or the Chernoff bounds to bound the deviation from the expected total runtime for a single machine, and conclude from that on the makespan.

    (For the Chernoff bounds, you may assume that it also holds if the variables X_i satisfy $X_i \in [0, 1]$ instead of $X_i \in \{0, 1\}$. For that to hold, you may have to renormalize the X_i's if, for example, X_i is defined as $X_i \in \{0, t_i\}$ for some $t_i \in \mathbb{N}$.)