Johannes Blömer
Jan Bobolz, Gennadij Liske

14. April 2016
submission due: 25. April 2016, 15:30

Complexity Theory

SS 2016

Homework 1

Exercise 1 (8 points)**:**
Consider the language $L = \{w \in \{0,1\}^* \mid \exists z \in \{0,1\}^* : w = zz^R\}$, where z^R is the reverse of z (i.e. if $z = z_1 \dots z_n$, then $z^R = z_n \dots z_1$).

a) Describe a 2-tape DTM that decides L in time $\mathcal{O}(n)$.

b) Describe a 1-tape DTM that decides L. What is its runtime?

It is not necessary to describe the DTM formally as a 7-tuple, but be precise in your description.
(*Note* that, as briefly mentioned in the lecture, L can be used to show $\mathbf{DTIME}_1(n) \neq \mathbf{DTIME}_2(n)$.)

Exercise 2 (4 points)**:**
Consider Dijkstra's algorithm for finding shortest paths. Its input is a directed graph $G = (V, E)$ with edge costs $c : E \to \mathbb{N}_0$ and $s, t \in V$. It outputs the length $\delta(s,t)$ of a shortest path from s to t.

```
δ(v) ← ∞ ∀v ∈ V                          ▷ all nodes initially unreachable
δ(s) ← 0                                 ▷ s has distance 0 from itself
Q ← V
while Q ≠ ∅ do
    u ← argmin_{v∈Q}(δ(v))               ▷ remove closest node remaining
    Q ← Q \ {u}
    for all Neighbors v of u do          ▷ update all neighbors' distances
        δ(v) ← min{δ(u) + c(u,v), δ(v)}
    end for
end while
return δ(t)
```

a) How much space does Dijkstra's algorithm use asymptotically?

b) Suppose the input V, E, c, s, t is supplied to the algorithm on a special read-only tape. How much (additional) writable memory does Dijkstra's algorithm require?

Advice: the answers to a) and b) should not the same (be more precise!)

Exercise 3 (4 points)**:**
In the lecture, we have seen an algorithm to decide $TQBF$. It requires polynomial space. What is its runtime in Θ notation?

Exercise 4 (8 points)**:**
Prove that if $L \in \mathbf{P}$, then $L^* \in \mathbf{P}$.
Hint: On input $x_1 \dots x_n$, use dynamic programming to build a table A, where $A[i,j]$ indicates whether or not $x_i \dots x_j \in L^*$.