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Complexity Theory

SS 2016

Class Handout 10

Exercise 1:
Show that $\mathbf{L} \subsetneq \mathbf{PSPACE}$.

Exercise 2:
Show that for any two real numbers $0 < \epsilon_1 < \epsilon_2$,

$$\mathbf{DSPACE}\left(n^{\epsilon_1}\right) \subsetneq \mathbf{DSPACE}\left(n^{\epsilon_2}\right) \quad .$$

You may use that $\lfloor n^c \rfloor$ is space-constructible for any $c \in \mathbb{Q}$.

Exercise 3:
Show Theorem 5.4, i.e. there is a universal Turing machine U that can simulate a $s(n)$ space DTM M in space $c \cdot \big(|\langle M \rangle| + s(n)\big)$ for some constant c.

a) What is (a well-formed) input to the universal Turing machine U?

b) What is $L(U)$?

c) Can you argue for a tighter space bound?

Exercise 4:
Consider the witness-based definition of **NL** from Homework 9. There, we defined a Turing machine with an additional witness tape whose head cannot move left. We have seen that a language $L \subseteq \{0,1\}^*$ is in **NL** if and only if there exists a *log space* deterministic Turing machine M with input and witness tape and a polynomial p such that

$$L \quad = \quad \left\{x \in \{0,1\}^* \;\middle|\; \exists z \in \{0,1\}^{p(|x|)} : (x,z) \in L(M)\right\} \quad .$$

We now consider the class co-**NL**.

a) Show that a language $L' \subseteq \{0,1\}^*$ is in co-**NL** if and only if there exists a *log space* deterministic Turing machine M' with input and witness tape and a polynomial p such that

$$L' \quad = \quad \left\{x \in \{0,1\}^* \;\middle|\; \forall z \in \{0,1\}^{p(|x|)} : (x,z) \in L(M')\right\} \quad .$$