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# Complexity Theory

SS 2016

Class Handout 3

**Exercise 1:**
Prove the statement of Corollary 2.32 from the lecture:

$$\mathbf{PSPACE} = \mathbf{NPSPACE} \ .$$

**Exercise 2:**
We have shown that TQBF $\in \mathbf{DSPACE}(n)$ and that TQBF is **PSPACE**-complete. Can we deduce that for every language $L \in \mathbf{PSPACE}$ it holds $L \in \mathbf{DSPACE}(n)$, that is $\mathbf{DSPACE}(n) = \mathbf{PSPACE}$?

**Exercise 3:**
Consider our current definition of space constructibility. Is the function

$$\begin{aligned} f : \mathbb{N} &\to \mathbb{N} \\ n &\mapsto \lfloor \log(n) \rfloor \end{aligned}$$

space constructible according to this definition?
Prove furthermore that if $f$ and $g$ are space constructible, then the following functions are space constructible too:

$$f + g, \qquad f \cdot g, \qquad f \circ g \ .$$

**Exercise 4:**
Prove that the language

$$\text{3SAT} = \{\langle \phi \rangle \mid \phi \text{ is a satisfiable 3CNF-formula}\}$$

is **NP**-complete. At first argue that 3SAT $\in \mathbf{NP}$. In order to prove that any language in **NP** is polynomial time reducible to 3SAT proceed as follows:

- Consider the Boolean formula constructed in the Cook-Levin Theorem and show that this formula can be transformed into an equivalent CNF-formula of appropriate size.
- Show that every CNF-formula can be converted into one of appropriate size with three literals per clause (using further variables).