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Complexity Theory

SS 2016

Class Handout 4

Exercise 1:
We write $A \leq_{sp} B$ if there is a polynomial *space* reduction of A to B, i.e. a function $f : \{0,1\}^* \to \{0,1\}^*$ that can be computed by a polynomial *space* Turing machine such that $w \in A \Leftrightarrow f(w) \in B$ for all $w \in \{0,1\}^*$.

a) Let $L, L' \in \mathbf{PSPACE}$ be arbitrary and $L' \neq \emptyset$ and $L' \neq \{0,1\}^*$. Prove that it holds $L \leq_{sp} L'$.

b) Where did you need that $L' \neq \emptyset$ and $L' \neq \{0,1\}^*$?

c) Why it does not make sense to define **PSPACE**-completeness over polynomial *space* reductions?

Exercise 2:
Prove for two languages $A, B \subseteq \{0,1\}^*$:

a) If $B \in \mathbf{NP}$ and $A \leq_p B$, then $A \in \mathbf{NP}$.

b) If B is **NP**-complete and $A =_p B$ (i.e. $A \leq_p B$ and $B \leq_p A$), then A is **NP**-complete.

c) If A is **NP**-complete, then the complement $\bar{A}$ of A is co-**NP**-complete.

Exercise 3:
Consider the **NP**-complete languages Clique and

$$\text{IndSet} = \left\{ \langle G, k \rangle \;\middle|\; \begin{array}{c} G = (V, E) \text{ is an undirected graph and} \\ \text{there exists a subset } U \subseteq V \text{ with } |U| = k \\ \text{such that no two nodes in } U \text{ are connected by an edge.} \end{array} \right\} .$$

Show by reduction:

$$\text{Clique} \leq_p \text{IndSet} .$$