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30. May 2016

Complexity Theory

SS 2016

Class Handout 7

Exercise 1:
Show that for any *bounded* polynomial time computable function $H : \mathbb{N} \to \mathbb{N}$, it holds that $SAT_H \in \mathbf{NPC}$.

Exercise 2:

a) Argue that all regular languages are in $\mathbf{L}$.

b) Consider the following grammar $G = (T, N, P, S)$ for correctly nested parentheses: $T = \{(,)\}, N = \{S\}$ and

$$P = \{S ::= (\ S\ ), S ::= S\ S, S ::= \epsilon\}.$$

Show that $L(G) \in \mathbf{L}$.

Exercise 3:
For two Boolean formulas ϕ, ϕ' over variables $X_1, \ldots, X_n$, we write $\phi \approx \phi'$ if they are semantically equivalent, i.e. $\forall (x_1, \ldots, x_n) \in \{0,1\}^n : \ \phi(x_1, \ldots, x_n) = \phi'(x_1, \ldots, x_n)$. The *length* $|\phi|$ of a formula ϕ is the number of Boolean operators in ϕ. ϕ is *minimal* if no equivalent formula is shorter, i.e. for all ϕ' with $\phi' \approx \phi$ it holds that $|\phi'| \geq |\phi|$.
We define

$$MINBOOL := \{\langle \phi \rangle \mid \phi \text{ is a minimal Boolean formula}\}.$$

a) Why is the following argument for the claim that $MINBOOL \in \text{co-}\mathbf{NP}$ not convincing? *We show that $\overline{MINBOOL} \in \mathbf{NP}$. $\overline{MINBOOL}$ is (essentially) the language of Boolean formulas ϕ for which a shorter equivalent formula exists. Given a formula ϕ, an NTM can nondeterministically guess a formula ϕ' of size $k < |\phi|$. If ϕ' is equivalent to ϕ, we accept (otherwise, we reject). Hence ϕ is accepted if and only if $\phi \in \overline{MINBOOL}$.*

b) Show that if $\mathbf{P} = \mathbf{NP}$ then $MINBOOL \in \mathbf{P}$.