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Randomized Algorithms

SS 2018

Homework Assignment 1

Problem 1:
Suppose that we pick $k \leq n$ out of n chairs at some round table uniformly at random and assign them to k people. What is the expected number of people that do not have a direct table neighbor? Hint: use indicator variables and the linearity of expectation.

Problem 2:
A ship enters some harbor and 40 sailors leave it to enjoy themselves. In the night, they all come back, but they are so drunk that each of them picks a random cabin out of the 40 cabins uniformly and independently at random. What is the expected number of sailors that fall asleep in their own cabin?

Problem 3:
Consider a random experiment Ω that can be represented as $\Omega = \Omega_1 \times \Omega_2$ with probability distributions $p_1 : \Omega_1 \to [0,1]$ and $p_2 : \Omega_2 \to [0,1]$ and the property that $\Pr[w] = p_1(w_1) \cdot p_2(w_2)$ for all $w = (w_1, w_2) \in \Omega$. Show that then for any two events $A_1 \subseteq \Omega_1$ and $A_2 \subseteq \Omega_2$ it holds for $A_1' = A_1 \times \Omega_2$ and $A_2' = \Omega_1 \times A_2$ that

$$\Pr[A_1' \cap A_2'] = \Pr[A_1'] \cdot \Pr[A_2'] = Pr[A_1] \cdot \Pr[A_2] \ .$$

Problem 4:
Prove Theorem 1.4.