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Paderborn, December 8, 2017
Submission: December 15, 2017

Fundamental Algorithms

WS 2017

Exercise Sheet 9

Exercise 1:
Prove Lemma 6.12, which states the following. Let (G, s, t, c) be a flow network and let f be a network flow in G. Then it holds for all $X, Y, Z \subseteq V$:

a) $f(X, X) = 0$

b) $f(X, Y) = -f(Y, X)$

c) If $X \cap Y = \emptyset$ then $f(X \cup Y, Z) = f(X, Z) + f(Y, Z)$ and $f(Z, X \cup Y) = f(Z, X) + f(Z, Y)$

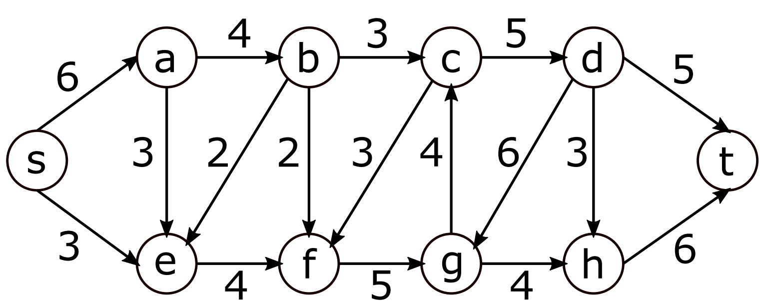
Exercise 2:
Prove Lemma 6.15, which states the following. Let (G, s, t, c) be a flow network and f be a flow in G. Let (S, T) be a cut of G. Then

$$f(S, T) = |f|.$$

In particular, also

$$|f| = f(s, V - s) = f(V - t, t).$$

Exercise 3:
Execute the Ford-Fulkerson Algorithm on the following flow network:



Before and after each iteration of the while-loop, present G with the current flow f, the residual network G_f, and the augmenting path that will be used in the next iteration. Finally, give a cut of G whose capacity equals the value of the maximum flow.

Exercise 4:
Suppose that, in addition to edge capacities, a flow network has *vertex capacities*. That is, each vertex v has a limit $l(v)$ on how much flow can pass through v. Show how to transform a flow network (G, s, t, c) with vertex capacities into an equivalent flow network (G', s', t', c') without vertex capacities such that a maximum flow in G' has the same value as a maximum flow in G. How many vertices and edges does G' have?