Tiara: A Self-Stabilizing Deterministic Skip List and Skip Graph

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Abstract

We present *Tiara* — a self-stabilizing peer-to-peer network maintenance algorithm. Tiara is truly deterministic which allows it to achieve exact performance bounds. Tiara allows logarithmic searches and topology updates. It is based on a novel *sparse 0-1 skip list*. We then describe its extension to a ringed structure and to a skip-graph.

Key words: Peer-to-peer networks, overlay networks, self-stabilization.

1. Introduction

Due to the rise in popularity of peer-to-peer systems, dynamic overlay networks have recently received a lot of attention. An overlay network is a logical network formed by its participants over a wired or wireless routing network. The number of computers and users in such a network may reach millions. Therefore, research in this area has focused on improving scalability and efficiency of overlay networks [1, 2, 3, 4, 5, 6, 7, 8, 9]. Two usual optimization properties are the speed of searching for items in the network and the speed of topology updates. Another popular parameter is expansion. It measures how robust a network is to worst-case faults and how many requests it can route in parallel without creating high congestion.

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Several popular designs of peer-to-peer networks are based on a sorted list or a ring [2, 3, 5, 9]. To decrease the network diameter and improve the search speed, the peers maintain a set of shortcut links to the nodes progressively further away. A skip list [10, 11] uses these shortcut links to build a a balanced tree over such a sorted list. Such construction enables searches and topology updates whose time is proportional to the logarithm of the number of nodes in the network. A skip graph [2, 3, 5] is an extension of a skip list that preserves the search and topology update properties of a skip list while significantly increasing its expansion.

In open peer-to-peer systems, participants may frequently enter and leave the overlay network either voluntarily or due to failures. In a peer-to-peer system of large scale, faults and inconsistencies are the norm rather than as an exception. Moreover, interaction of such faults may leave the system in an unpredictable, possibly system-wide failure. Hence, overlay networks require mechanisms that continuously counter such disturbances.

Solutions presented in the research literature mostly focus on efficiency of the proposed structure while offering only ad hoc solutions to fault tolerance. Such simplistic ad hoc approaches that handle individual fault conditions do not adequately perform in case of unanticipated, complex or systemic failures. In practice, many peer-to-peer systems, such as KaZaA, Bittorrent and Kademlia, use heuristic methods in order to maintain their topology. One can argue that if nodes are randomly distributed, a sorted list or ring with a sufficient number of redundant connections will withstand a fault with high probability. However, the problem of generating high-quality trusted random numbers in a peer-to-peer system is complicated [12]. Moreover, it is known that an adversary can quickly degrade the randomness of the peer-to-peer system even if perfectly random numbers are reliably generated [13, 14, 15]. Furthermore, while a faulty state is improbable, it is not impossible. Failure to consider recovery from such states may lead to catastrophic consequences. Thus, some researchers [16, 17] argue that overlay network architects need to consider holistic approaches to fault tolerance and recovery, such as self-stabilization.

In this paper we present Tiara. It is a self-stabilizing deterministic skip-list. We extend Tiara to a deterministic skip graph. To the best of our knowledge Tiara is the first deterministic self-stabilizing skip list and skip graph presented in the literature.

Related literature. In the field of self-stabilization, researchers study al-

gorithms that are guaranteed to eventually converge to a desirable system state from an arbitrary initial configuration. The idea of self-stabilization in distributed computing first appeared in a classical paper by E.W. Dijkstra in 1974 [18] in which he looked at the problem of self-stabilization in a token ring. Since Dijktra's paper, self-stabilization has been studied in many contexts, including communication protocols, graph theory problems, termination detection, clock synchronization, and fault containment. For a survey see, e.g., [19, 20, 21].

Traditional self-stabilizing techniques assume fixed topology that is not under the control of the algorithm itself. Although, often quite sophisticated, generalized self-stabilizing algorithms are not directly applicable to peer to peer networks. Awerbuch and Varghese [22] showed that every local algorithm can be made self-stabilizing if all nodes keep a log of the state transitions until the current state. Since then several other methods have emerged including various local and global checking and correction techniques [23, 24, 25, 26, 27]. Time-adaptive techniques [28, 29, 30] as well as local stabilizers [31] have been presented which can recover any distributed algorithm in $O(f)$ time depending only on the number f of faults. The considered faults do not include topological changes. Super-stabilization [32] addresses a single change in topology as well as global state corruption. This change is topology is not under the control of the algorithm.

In the area of peer-to-peer networks the researchers mostly focused on construction and maintenance of scalable systems (e.g. [33, 34, 35, 36]). Chen and Chen [37] described a sophisticated algorithm to deal with a variety of faults. Their solution is not self-stabilizing. In the technical report about the Chord network [38], several techniques such as the weak stabilization protocol and the strong stabilization protocol are presented that allow Chord to recover quickly from various kinds of degenerate states which the authors call *pseudo-trees* or *loopy states*. However, recovery from arbitrary state is not considered. Similarly, there are a number of techniques to recover skip graphs from certain degenerate states [39].

Self-stabilizing solutions to a sorted list and ring networks are discussed in the literature. Specifically, Aspnes et al. [40] described an asynchronous algorithm which turns an initially weakly connected graph into a sorted list. Their algorithm is not self-stabilizing. In a follow-up paper [41], a self-stabilizing algorithm was given for the case that nodes initially have outdegree 1. Onus et al. [42] presented a synchronous algorithm that converts an arbitrary connected graph into a sorted list. Gall et al. [43] introduced a

communication model that adequately captures realistic process contention and performance bottlenecks. In their paper, Gall et al. elaborated on the algorithms of Onus et al., presented two updated algorithms together with analysis of their distributed runtime in various settings. Shaker and Reeves [44] described a distributed algorithm for forming a directed ring network topology.

A few self-stabilization results have also been shown for more scalable networks. Dolev et al. [45] describe a self-stabilizing intrusion-tolerant overlay network. Dolev and Kat [46] introduce the HyperTree and use it as a basis for their self-stabilizing peer-to-peer system. Jacob et al. [47] present a self-stabilizing version of the randomized skip graph in [2] and Jacob et al. [48] demonstrate that the Delaunay graph has a self-stabilizing algorithm as well.

There are a number of studies on self-stabilizing ring or tree formation within some overlay network. Specifically, Hérault et al. [49] describe a spanning tree formation algorithm for overlay networks. Cramer and Fuhrmann [50] demonstrated that a ring-based overlay network ISPRP is self-stabilizing in certain cases. Caron et al. [51] described a snap-stabilizing prefix tree for peer-to-peer systems. Bianchi et al. [52] presented a stabilizing search tree for overlay networks optimized for content filters.

Our contribution. In this paper we present Tiara. It stabilizes a novel distributed skip list. Specifically, we demonstrate a self-stabilizing algorithm for a sorted list and then show how to extend it to a self-stabilizing algorithm for a skip list. Tiara can construct these structures without any knowledge of global network parameters such as the number of nodes in the system: each node utilizes only the information available about its immediate neighbors. Moreover, Tiara preserves connectivity during stabilization. We rigorously prove Tiara correct in an asynchronous shared-register communication model. We estimate Tiara's stabilization time to be linear with respect to the system size.

We show how Tiara can be extended to form ring structures and prove the correctness of the resulting algorithm. This demonstrates the applicability of Tiara to a number of peer-to-peer algorithms, such as Chord, that utilize circular structures. On the basis of Tiara we develop an algorithm that maintains a deterministic skip graph. We prove that the search cost in this skip graph is logarithmic while the update cost is polylogarithmic. While the expansion of a skip list is $\Theta(1/|N|)$ (where N is the set of nodes), it turns

out that the expansion of the skip graph is $\Omega(1/|N|^{1/\log 3})$.

We extend Tiara to handle voluntary joins and leaves and discuss other practical implementation concerns.

Organization of the paper. First, we introduce our computational model. Then, we describe a self-stabilizing algorithm for the sorted list and formally prove it correct. We then extend it to a ring structure and a skip graph. We complete the paper with future research directions and open problems.

2. Model and Skip List Definitions

Model. A peer-to-peer system consists of a set N of processes. Each process has a unique integer identifier. A process contains a set of variables and actions. An action has the form $\langle name \rangle : \langle guard \rangle \longrightarrow \langle command \rangle$. In this form, *name* is a label, *guard* is a Boolean predicate over the variables of the process and *command* is a sequence assigning new values to the variables of the process. For each pair of processes a and b, we define a Boolean variable (a, b) that is shared between them. Two processes a and b are *neighbors* if this variable is **true**. The *neighborhood* of a process a is defined as the set of all of its neighbors. Sets of neighbors may be maintained on different *levels*. A neighborhood of process a at level i is denoted as $a.i.NB$. The *right neighborhood* of a, denoted $a.i.R$, is the set of neighbors of a with identifiers larger than a. Similarly, the *left neighborhood* of a, denoted $a.i.L$, are a's neighbors with smaller identifiers. Naturally, the union of $a.i.R$ and $a.i.L$ is $a.i.NB$.

We use quantified boolean expressions of the form $(E : range : body)$ (see [53, Chapter 2]) to specify predicates; where E is a universal or existential quantifier followed by a bound variable, *range* is a predicate that determines the values of the bound variable while *body* is a predicate based the bound variable. In effect the two predicates form a conjunction that determines the values of the bound variable. For example, the predicate $(\forall a : a > 0 : a \neq b)$ is true if for all $a > 0$ it holds that $a \neq b$. If the range is omitted, the predicate applies to all values of the bound variable. We use similar notation for sets. For example, $a.i.R \equiv \{b : b \in a.i.NB : b > a\}$ and $a.i.L \equiv \{b : b \in b.i.NB : b < a\}$.

When describing a link, we always state the smaller identifier first. That is, a is less than b in (a, b). Two processes a and b are *consequent* if there is no process c whose identifier is between a and b. That is, $\mathbf{cnsq}(a, b) \equiv (\forall c ::$

$(c < a) \vee (b < c)$). The *length* of a link (a, b) is the number of processes c such that $a < c < b$. By this definition the length of a link that connects consequent processes is zero.

A *system state* is an assignment of a value to each variable of every process of the system. An action is *enabled* in some state if its guard is **true** at this state. A *computation* is a maximal sequence of states such that for each state s_i, the next state s_{i+1} is obtained by executing the command of an action that is enabled in s_i. This definition disallows the overlap of action executions. That is, an action execution is *atomic*. The execution of a single action is a *step*. *Maximality* of a computation means that the computation is infinite or it terminates in a state where none of the actions are enabled. In the latter case, such a state is a *fixpoint*. In a computation we assume the action execution to be *weakly fair*. That is, if an action is enabled in all but finitely many states of an infinite computation then this action is executed infinitely often. This defines an *asynchronous* program execution model.

A state *conforms* to a predicate if this predicate is **true** in this state; otherwise the state *violates* the predicate. By this definition every state conforms to the predicate **true** and no state conforms to **false**. Let T and U be predicates over the state of the program. Predicate T is *closed* with respect to the program actions if every state of the computation that starts in a state conforming to T also conforms to T. Predicate T *converges* to U if T and U are closed and any computation starting from a state conforming to T contains a state conforming to U. The program *stabilizes* to T if **true** converges to T.

While most of our program model is fairly conventional, we would like to draw the reader's attention to our way of modeling overlay network link management. If a process updates its neighborhood, the change also affects its neighbors. For example, if process a adds b to its neighborhood by creating a link (a, b), this also means that a is atomically added to b's neighborhood. If a removes b from its neighborhood, then also a is removed from b's neighborhood. We discuss ways of implementing such atomic action in the future work section.

Peer-to-peer and skip list structures. A peer-to-peer network consists of a set of nodes with distinct identities. In order to allow efficient searching for nodes, the nodes can be arranged in a sorted list or a ring. The search time on a simple sorted list or ring is proportional to the network size. To increase the search speed, shortcut links are needed. In a skip-list these links

are added in levels. The lowest level is the sorted list of all nodes in the system. A node *belongs* to a higher level if it contains links at this level.

A deterministic skip-list is defined recursively by levels. The 0-level is a sorted list of all identifiers of the system. In each level i of a *k-ℓ skip list*, a node u has a link to node v if and only if u and v are between k and ℓ hops away at level $i-1$.

For example, a 0-1 skip list is such that if u and v are neighbors at level i, then, at level $i-1$, they are either neighbors or two hops away from each other. A 0-1 skip list is particularly suitable for the communication register model that we use in this paper since each process can determine the identities of its potential neighbors at level i by reading the states of its neighbors at level $i-1$.

However, as defined, a 0-1 skip list may not be useful as a peer-to-peer structure. Indeed, this definition admits a graph where every node links to its lowest-level neighbors at all other levels. Therefore, we place a restriction on such lists. A *sparse 0-1 skip list* is a skip list such that if nodes v and w are neighbors of node u at level i then either v or w is not a neighbor of u at level $i-1$. By this definition, out of three consequent neighbor nodes at level $i-1$, at most two may be present at level i.

If $k > 0$ in a k-ℓ skip list, as well as in a sparse 0-1 skip list, a fraction of all nodes at level $i-1$ is not present at level i. Therefore, such skip list diameter is logarithmic with respect to the network size. However, a skip list may not be convenient for concurrent searches or robust to node failures. Indeed, the crash of any top-level node disconnects the whole system.

In a 0-1 skip list, some nodes that have links at level $i-1$ do not have links at level i or higher. To increase the robustness and concurrency of the overlay network, these nodes may be connected in an alternative list at level i'. Specifically, in a *0-1 skip graph* node v has a link to node v at level i' if and only if v is present at level $i-1$, absent at level i and v and there is such node between v and u at level $i-1$. Note that this definition is recursive. That is with transition to higher level, every list is split into two higher level lists. In other words, there is a single list at level 0, there are two at level 1, four at level 2 and so on.

In a linear structure, the mid-range nodes tend to be used more often than edge nodes for navigation and searches. To eliminate this midrange node contention, ring structures such as Chord [9] employ *wraparound* links between the first and the last node at each level. Using the wraparound link the search can efficiently jump from one edge of the system to the other.

See Figure 1 for an example of a skip graph with wraparound links based on a sparse 0-1 skip list.

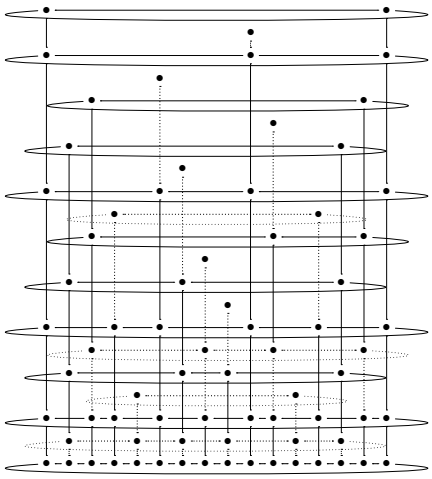


Figure 1: An example of 0-1 skip graph with wraparound links.

3. Core Tiara

Outline. In its core, Tiara contains two components: the bottom component (b-Tiara) that maintains the processes at the lowest level in sorted order and the skip-list component (s-Tiara) that constructs the higher levels of Tiara.

We present the components and prove them correct bottom up starting with b-Tiara. In the correctness proof, we use the classic stabilization by levels [54] argument: we prove that the lower level stabilizes to a certain predicate; then, in the proof of stabilization of the upper level, we assume that the lower level predicate already holds; this ensures the stabilization of the whole system provided that the stabilization of the lower level does not depend on actions of the upper one.

However, the operation of b-Tiara and s-Tiara is interdependent. s-Tiara relies on b-Tiara to sort the lowest level while s-Tiara may append links to the bottom level so that these links are not discarded and the overall connectivity of the system is preserved.

Yet, we are still able to apply the classic stabilization proof techniques by separating the proof of b-Tiara into two parts: grow and trim. The grow part of b-Tiara links processes whose identifiers are closer to each other

process u
variables
$u.0.NB$ — set of neighbor processes of u.
shortcuts
$u.0.L \equiv \{z : z \in u.0.NB : z < u\},\ u.0.R \equiv \{z : z \in u.0.NB : z > u\}$
actions
grow right: $(s \in u.0.R) \wedge (t \in s.0.L) \wedge (t \notin u.0.NB) \longrightarrow$
$u.0.NB := u.0.NB \cup \{t\}$
trim right: $(s, t \in u.0.R) \wedge (t \in s.0.L) \wedge (\forall z : z \in u.0.R : z \leq s) \wedge (\forall z : z \in s.0.L : z \geq u) \longrightarrow$
$u.0.NB := u.0.NB \setminus \{s\}$
grow left and *trim left* are similar

Figure 2: The bottom component of Tiara (b-Tiara).

than their existing neighbors. The trim part of b-Tiara removes the links to the processes that are further away once the closer ones are connected. We prove that b-Tiara stabilizes to a state where the consecutive processes are connected.

Since s-Tiara only adds links to the bottom level but does not remove them, s-Tiara does not affect this stabilization. We prove stabilization of s-Tiara assuming that consecutive processes are connected at the bottom level.

Once s-Tiara stabilizes, it does not add links to the bottom level any more. We assume that s-Tiara is stable and prove that b-Tiara trims every link that connects non-consequent process. This completes the stabilization of core Tiara.

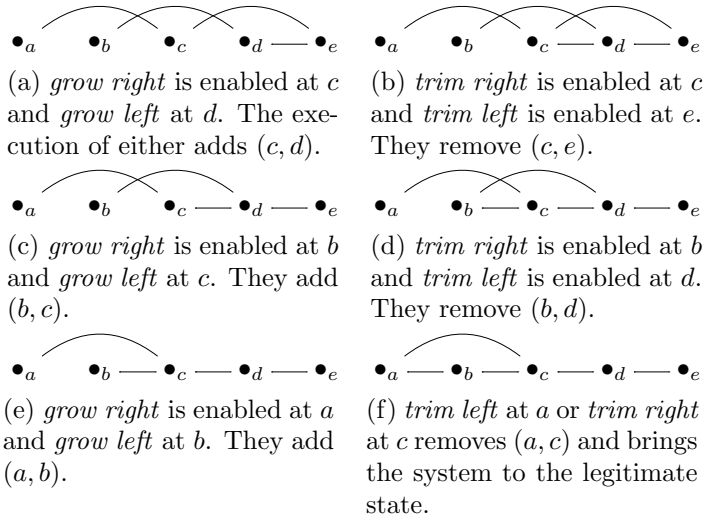
3.1. The Bottom Component of Tiara (b-Tiara) and Stabilization of Grow

Description. The objective of b-Tiara is to transform the system into a linear graph with the processes sorted according to their identifiers. The algorithm for b-Tiara is shown in Figure 2. The only variables that b-Tiara manipulates are the neighbor sets for each process u — $u.0.NB$. The *right neighborhood* of u, denoted $u.0.R$ is a subset of $u.0.NB$ with the identifiers greater than u. Since $u.0.R$ can be computed from $u.0.NB$ as necessary, $u.0.R$ is not an independent variable but a convenient shortcut. The *left neighborhood* $u.0.L$ is defined similarly.

Each process u has two pairs of actions: *grow* and *trim* that operate to the right an to the left of u. Action *grow right* is enabled if u discovers that its right neighbor s has a left neighbor t that is not a neighbor of u. In this case u adds t to its neighborhood. That is, u adds a link (u, t) to the graph. Even though u is the left neighbor of s, t may be either to the left or to the

right of u. That is $t < u$ or $t > u$. Regardless of this relation, u connects to t. Action *grow left* operates similarly in the opposite direction.

Action *trim right* eliminates extraneous links from the graph. This action removes link (u, s) if u has a neighbor s that satisfies the following properties. The guard for *trim right* stipulates that there has to be another process t that is a neighbor of both u and s. Hence, if (u, s) is removed the connectivity of the graph is preserved. Also, all right neighbors of u must be smaller than or equal to s and all left neighbors of s are greater than or equal to u. The latter condition is necessary to break symmetry and prevent continuous growing and trimming of the same link. Action *trim left* operates similarly in the opposite direction. We show an example operation of b-Tiara in Figure 3.



(a) *grow right* is enabled at c and *grow left* at d. The execution of either adds (c, d).

(b) *trim right* is enabled at c and *trim left* is enabled at e. They remove (c, e).

(c) *grow right* is enabled at b and *grow left* at c. They add (b, c).

(d) *trim right* is enabled at b and *trim left* is enabled at d. They remove (b, d).

(e) *grow right* is enabled at a and *grow left* at b. They add (a, b).

(f) *trim left* at a or *trim right* at c removes (a, c) and brings the system to the legitimate state.

Figure 3: Example computation of *b-Tiara*. The processes are listed in increasing order of their identifiers.

Proof of stabilization of grow part of b-Tiara. Denote by $B(N) = (N, E(B(N)))$ the graph that is induced by the processes of the system and the links of b-Tiara. We define the following predicate: $\mathcal{GI} \equiv (\forall a, b : a, b \in N : \mathbf{cnsq}(a, b) \Rightarrow (a, b) \in E(B(N)))$. That is, $\mathcal{GI}$ states that two consequent processes are also neighbors.

Lemma 1. *If a computation of b-Tiara starts from a state where $B(N)$ is connected, it is connected in every state of this computation.*

Proof. The actions of b-Tiara do not disconnect $B(N)$. Indeed, the actions that remove links are *trim right* and *trim left*. Consider *trim right*. It removes a link (a, b) if there exists a node c such that there are links (a, c) and (c, b). Thus, the removal of (a, b) does not disconnect the graph. The argument for *trim left* is similar. □

Lemma 2. *If a computation of b-Tiara starts from a state where $B(N)$ is connected, b-Tiara stabilizes to $\mathcal{GI}$.*

Proof. To prove the lemma we need to show that (i) $\mathcal{GI}$ is closed under the execution of the actions of b-Tiara and (ii) regardless of the initial state, every computation contains a state satisfying $\mathcal{GI}$. Let us consider closure first. The *grow* actions may not violate $\mathcal{GI}$ as they only add links. The *trim* action may affect $\mathcal{GI}$ by disconnecting two processes a and b. However, *trim right*, which removes link (a, b), is only enabled at process a if there is a process c such that $a < c < b$. Therefore, if a and b are consequent, *trim right* is disabled. The reasoning is similar for *trim left*. Hence the closure.

Let us consider convergence now. Assume that there are two consequent processes a and b that are not neighbors. That is $b \notin a.0.NB$. Since the graph itself is connected, there is a path ρ between a and b. If there are multiple paths, we shall consider the shortest one. Let the length of ρ be the sum of the lengths of its constituent links. We use variant function [55] technique to prove convergence where the length of ρ is used as this function. We demonstrate that this length eventually decreases to zero which implies convergence.

Indeed, the execution of a *trim* action does not change the length of ρ. The execution of any of the *grow* actions does not increase the length of ρ. Path ρ must contain at least one segment d, e, f such that both d and f are either smaller than e or larger than e. In this case *grow right*, or respectively, *grow left*, is enabled in both d and f. The execution of this action decreases the length of the path. Hence, throughout the computation, the length of ρ decreases until it is zero and a and b are neighbors. The lemma follows. □

3.2. The Skip List Component of Tiara (s-Tiara)

Description. The objective of s-Tiara is to establish a skip list on top of the linearized graph created by b-Tiara. The structure maintained by s-Tiara is a *sparse 0-1 skip list*. At each level i, node u maintains a set of neighbors $u.i.NB$. Out of this set, the rightmost and leftmost neighbors are defined as

right and left skip links: $u.i.rs$ and $u.i.ls$. A node may not have a right or left skip link at some level if it is on either end of the list.

We denote right and left skip list neighbors of u at level $i-1$ as v and x respectively. Nodes w and y are respectively right and left neighbors of v and x at the same level. We illustrate this notation in Figure 4 as we will be using it extensively throughout the correctness proof of the algorithm.

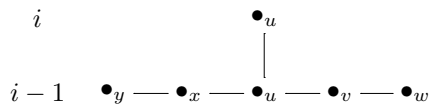
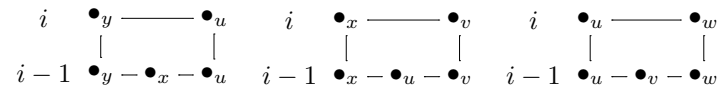


Figure 4: Aliases for neighbors of u in *s-Tiara*. $v \equiv u.(i-1).rs$, $w \equiv v.(i-1).rs$, $x \equiv u.(i-1).ls$, and $y \equiv x.(i-1).ls$, where $u.i.rs$ and $u.i.ls$ are right and left skip-list neighbors of u at level i, respectively.

If both nodes u and v exist at level i and $u.i.rs = v$ then this link is a 0-*skip link*. If u and w exist at level i and $u.i.rs = w$, then this link is a 1-*skip link*. A process that exists at level $i-1$ is *up* if it also exists at level i; it is *down* otherwise. u, v and w form a *cage* if $u.i.rs$ contains w and v is down. The middle process v is *inside* the cage. Refer to Figure 5 for the illustration of the concept of a cage. The sparse 0-1 skip list has two rules of organization. First, all links are either 0- or 1-skip links. Second, if a node is on level i and it is not at the end of the list on level $i-1$ then at least one of its links is a 1-skip link.



(a) u is adjacent to the cage on the left.

(b) u is inside the cage.

(c) u is adjacent to the cage on the right.

Figure 5: Possible cages with respect to node u.

The algorithm is shown in Figure 6. As before, to simplify the presentation we introduce a few shortcuts. Sets $u.i.R$ and $u.i.L$ are the subsets of $u.i.NB$ that contain the identifiers of u's neighbors with respectively higher and lower identifiers than u. We define $u.i.rs$ to be the neighbor with the link of the smallest length among $u.i.R$. To put another way, $u.i.rs$ connects to u's right neighbor with the smallest identifier. Note that $u.i.rs$ is $\perp$ if $u.i.R$ is empty. Shortcut $u.i.ls$ is defined similarly.

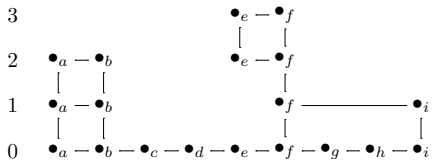
process u
parameter $i \geq 0$: **integer** — level of the skip list
variables
$u.i.NB$ — set of neighbor processes of u at level i
shortcuts
$v \equiv u.(i-1).rs,\ w \equiv v.(i-1).rs,\ x \equiv u.(i-1).ls,\ y \equiv x.(i-1).ls$
$u.i.R \equiv \{z : z \in u.i.NB : z > u\},\ u.i.L \equiv \{z : z \in u.i.NB : z < u\}$
$u.i.rs \equiv \begin{cases} s \in u.i.R : (\forall t : t \in u.i.R : t \geq s), & \text{if } u.i.R \neq \varnothing \\ \bot, & \text{otherwise} \end{cases}$
$u.i.ls$ is defined similarly
$\mathbf{exists}(z, i) \equiv ((z \neq \bot) \wedge (z.i.NB \neq \varnothing))$
$\mathbf{valid}(u, i) \equiv ((\mathbf{exists}(x, i) \wedge \mathbf{exists}(u, i) \wedge \mathbf{exists}(v, i)) \Rightarrow$
$(((u.i.ls = y) \vee (u.i.ls = x) \vee (u.i.ls = \bot)) \wedge (u.i.rs = w)) \vee$
$(((u.i.rs = v) \vee (u.i.rs = w) \vee (u.i.rs = \bot)) \wedge (u.i.ls = y)) \vee$
$((u.i.ls = \bot) \wedge (u.i.rs = \bot)))$
actions for $i > 0$
upgrade right: $\mathbf{valid}(u, i) \wedge \neg\mathbf{exists}(v, i) \wedge (v \neq \bot) \wedge (w \neq \bot) \wedge (u.i.rs \neq w) \longrightarrow$
$u.i.NB := u.i.NB \cup \{w\}$
upgrade left is similar
bridge right: $\mathbf{valid}(u, i) \wedge \mathbf{exists}(u, i) \wedge \mathbf{exists}(v, i) \wedge (u.i.rs \neq v) \longrightarrow$
$u.i.NB := u.i.NB \cup \{v\}$
bridge left is similar
prune: $\mathbf{valid}(u, i) \wedge \mathbf{exists}(u, i) \wedge (u.i.NB \neq \{u.i.rs, u.i.ls\}) \longrightarrow$
$u.0.NB := u.0.NB \cup u.i.NB \setminus \{u.i.rs, u.i.ls\},$
$u.i.NB := \{u.i.rs, u.i.ls\}$
downgrade right: $\neg\mathbf{valid}(u, i) \wedge \neg((u.i.rs = v) \vee (u.i.rs = w) \vee (u.i.rs = \bot)) \longrightarrow$
$u.0.NB := u.0.NB \cup u.i.R,$
$u.i.R := \varnothing$
downgrade left is similar
downgrade center: $\neg\mathbf{valid}(u, i) \wedge \mathbf{exists}(x, i) \wedge \mathbf{exists}(u, i) \wedge \mathbf{exists}(v, i) \longrightarrow$
$u.0.NB := u.0.NB \cup u.i.NB,$
$u.i.NB := \varnothing$

Figure 6: The skip list component of Tiara (s-Tiara).

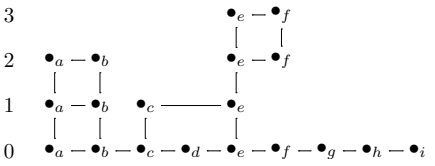
Predicate $\mathbf{exists}(z, i)$ is **true** if node z is present and if $z.i.NB$ is not empty. Node u may read only its immediate neighbor states. Thus, u may only invoke **exists** on its neighbors and itself. Observe that **exists** is defined to return **false** if it is invoked on a non-existent node. For example, if u is at the right end of the list at level i and u invokes $\mathbf{exists}(u.i.rs, i)$. In this case $\mathbf{exists}(u.i.rs, i)$ returns **false**. Predicate $\mathbf{valid}(u, i)$ captures the correct state of the system. Specifically, it states that if u exists at level i then the length of the skip links should not be more than 1 and either x or v does not exist at level i. The latter condition guarantees that at least one link of u is a 1-skip link.

The actions of s-Tiara are as follows. Action *upgrade right* establishes a link to w at level i if v is not up. That is, this link is a 1-skip link. Note that if either u or w are not not up, *upgrade right* brings them to level i.

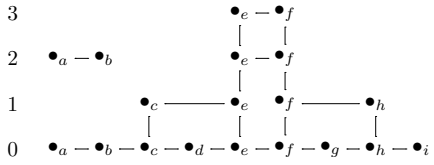
Action *upgrade left* operates similarly in the opposite direction. Actions *bridge right* and *left* establish 0-skip links if both nodes being connected are up. Action *prune* eliminates the links other than $u.i.rs$ and $u.i.ls$ from $u.i.NB$. In case the links are not 0- or 1-skip, action *downgrade right* completely removes the right neighborhood of u. Action *downgrade left* operates similarly. And the last action *downgrade center* eliminates three consecutive up nodes. This ensures that there cannot be two consecutive 0-skip links. An example computation of s-Tiara is shown in Figure 7.



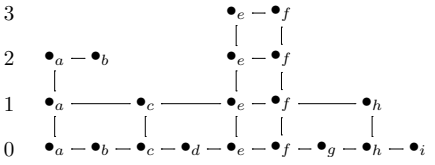
(a) Level 1. *downgrade right* is enabled at f, *downgrade left* is enabled at i and *upgrade left* is enabled at e. These actions remove (f, i) and add (e, c).



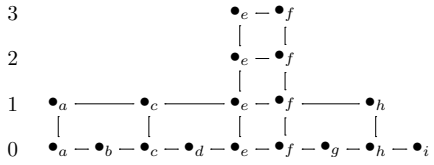
(b) Level 1. *downgrade center* is enabled at b, *upgrade right* is enabled f and *upgrade left* is enabled at h. These actions remove (a, b) and add (f, h).



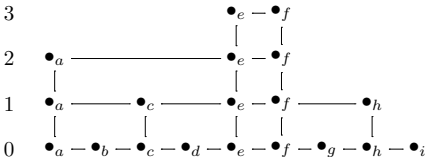
(c) Level 1. *upgrade right* is enabled at a, *upgrade left* is enabled at c, *bridge right* is enabled at e and *bridge left* is enabled at f. These actions add (a, c) and (e, f).



(d) Level 2. *downgrade right* is enabled at a and *downgrade left* is enabled at b. These actions remove (a, b).



(e) Level 2. *upgrade right* is enabled at a and *upgrade left* is enabled at e. These actions add (a, e).



(f) The system has reached a legitimate state.

Figure 7: *s-Tiara*. We list the processes in the increasing order of their identifiers. *b-Tiara* has stabilized to $\mathcal{GI}$. In each state we only mention the enabled actions that are relevant to the discussion. We do not illustrate the operation of *prune*.

Correctness proof. To prove correctness of s-Tiara, we use a method similar to convergence stair [56]. Specifically, we state a sequence of predicates on the level i of s-Tiara. We prove a series of lemmas where in each lemma we show that if the lower levels of s-Tiara as well as the preceding predicates in the level i at level i have stabilized, then level i of s-Tiara stabilizes to the next predicate in the sequence. The conjunction of these predicates implies the stabilization of level i of s-Tiara. We then use this fact as an inductive step in the convergence proof of stabilization of s-Tiara.

Before proceeding with the proof, we introduce notation and terminology we are going to use. Denote $S(N)$ the graph induced by the processes of the system as well as the links of b-Tiara and s-Tiara. Throughout the discussion we consider process u and its neighbors as defined in the description of s-Tiara. A node u is a *middle* node at level i if it has both left and right neighbors as well as at least one two-hop neighbor at level $i-1$. That is, $\mathbf{middle}(u,i) \equiv (\mathbf{exists}(v,i-1) \wedge \mathbf{exists}(x,i-1) \wedge (\mathbf{exists}(y,i-1) \vee \mathbf{exists}(w,i-1)))$.

The predicates to which s-Tiara stabilizes are as follows. The $\mathbf{good\_links}.i$ predicate states that process u connects to processes at most two hops away. Predicate $\mathbf{one\_links}.i$ enforces the rules of the 0-1 skip list. Specifically, it stipulates that u should either be inside the cage or should have adjacent cages to the left or to the right. Predicates $\mathbf{zero\_right\_links}.i$ and $\mathbf{zero\_left\_links}.i$ ensure that the 0-links are in place. That is, the processes that are consequent at level $i-1$ and are up, are also connected at level i. Predicate $\mathbf{only\_good\_links}.i$ states that the neighborhood of u does not have links other than rs and ls.

$$
\begin{aligned}
\mathbf{good\_links}.i \equiv\ & (\forall u :: \mathbf{exists}(u.i)\ \Rightarrow \\
& ((u.i.rs = v) \vee (u.i.rs = w) \vee (u.i.rs = \bot)) \wedge \\
& ((u.i.ls = y) \vee (u.i.rs = x) \vee (u.i.ls = \bot)))
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{one\_links}.i \equiv\ & (\forall u :: \mathbf{middle}(u, i-1)\ \Rightarrow \\
& (\neg\mathbf{exists}(u,i) \wedge (x.i.rs = v) \wedge (v.i.ls = x)) \vee \\
& (\neg\mathbf{exists}(v,i) \wedge (\neg\mathbf{exists}(w, i-1) \vee (u.i.rs = w))) \vee \\
& (\neg\mathbf{exists}(x,i) \wedge (\neg\mathbf{exists}(y, i-1) \vee (u.i.ls = y))))
\end{aligned}
$$

$$\mathbf{zero\_right\_links}.i \equiv (\forall u :: (\mathbf{exists}(u.i) \wedge \mathbf{exists}(v.i))\ \Rightarrow\ (u.i.rs = v))$$

$$\mathbf{zero\_left\_links}.i \equiv (\forall u :: (\mathbf{exists}(u.i) \wedge \mathbf{exists}(x.i))\ \Rightarrow\ (u.i.ls = x))$$

$$\mathbf{only\_good\_links}.i \equiv (\forall u :: \mathbf{exists}(u.i)\ \Rightarrow\ (u.i.NB = \{u.i.rs, u.i.ls\}))$$

Lemma 3. *Assuming that neighbor relations at level $i-1$ (i.e., $*.i-1.ls$ and $*.i-1.rs$) do not change throughout the computation, s-Tiara stabilizes to* **good_links**$.i$

Proof. In proving this and subsequent lemmas we show a stronger property of closure and convergence of the predicate for a particular process u. This implies the stabilization of the predicate for all u at the specified level.

Let us show closure first. Suppose that the neighbor relations at level $i-1$ do not change. Let us consider the actions and how they affect **good_links**$.i$. We start with the actions of u. Actions *upgrade right* and *bridge right* do not violate the predicate since they set $u.i.rs$ to v or w. A similar argument applies to *upgrade left* and *bridge left*. Action *prune* does not affect the predicate since it does not modify either $u.i.rs$ or $u.i.ls$. Neither do *downgrade right* and *downgrade left* since they respectively set $u.i.rs$ and $u.i.ls$ to $\perp$. Action *downgrade center* removes u from level i altogether and hence cannot violate the predicate. The nodes further than two hops away never connect to u. Hence the actions of other nodes cannot violate the predicate either.

Let us now address convergence. The predicate can be violated only if u is up. It is violated if either $u.i.rs$ or $u.i.ls$ points to a node other than u's one or two-hop neighbors. In this case either *downgrade right* or *downgrade left* are enabled that bring the links in compliance with the predicate. □

Lemma 4. *Assuming that neighbor relations at level $i-1$ do not change throughout the computation and* **good_links**$.i$ *is satisfied, s-Tiara stabilizes to* **one_links**$.i$

Proof. As a first step, we would like to make the following observation: once a cage is formed, it is never destroyed. For example, assume that u, v and w form a cage. The actions of u, and, similarly, w do not affect the i-level link (u, w). Also, if v is down, the only actions it can use to come up is *upgrade right* or *upgrade left*. However, both are disabled since u and w are up. This observation guarantees the closure of **one_links**$.i$.

Let us discuss convergence. We consider two cases: u is initially down and u is initially up and never goes down. If u is down, the only way, u can come up is through execution of *upgrade right* or *upgrade left* at u, w or y. In all cases, cages adjacent to u are formed and the predicate is satisfied. Moreover, if u is down, then *upgrade right* is enabled in x and *upgrade left* in v. Thus, if u does not come up before, then x or v execute these *upgrade* actions. In this case, a cage is formed with u inside. This satisfies the predicate as well.

Assume that u is up. If it ever goes down, the foregoing discussion applies. The only remaining case is if u stays up for the remainder of the computation. In a computation of b-Tiara in which the neighbor relations in level $i-1$ are stable, a node can come up only once. Indeed, a node comes up only if it forms a cage. Since a cage is never destroyed, the node never goes down. This means that a node can go down only once. Let us consider the state of the computation where u's neighbors x and v do not change their up and down position. Both x and v cannot be simultaneously up in this state, as it enables *downgrade center* at u. The execution of this action brings u down. However, we assumed that u stays up for the remainder of the computation. Thus, either x or v are down. Assume, without loss of generality, that v is down. If w does not exist at level $i-1$, **one_links**.i is satisfied. Assume that w exists. If link $u.i.rs = w$ is present, **one_links**.i is also satisfied. However, if it is not present, then *upgrade right* is enabled in u. Its execution establishes the link, forms a cage and satisfies the predicate. □

Lemma 5. *Assuming that neighbor relations at level $i-1$ do not change throughout the computation and* **good_links**.i *as well as* **one_links**.i *are satisfied, s-Tiara stabilizes to* **zero_left_links**.i *and* **zero_right_links**.i

Proof. We prove the lemma for **zero_right_links**.i only. The proof for **zero_left_links**.i is similar. Let us argue closure. If **one_links**.i is satisfied processes do not go up or down. Thus, the only actions that can be enabled are *bridge* and *prune*. The execution of either action maintains the validity of **zero_right_links**.i. Hence the closure.

Let us address convergence. The predicate is violated only if the neighbor processes u and v are both up and they do not have a link at level i. If **one_links**.i is satisfied, u forms a cage to its left (or u has no two-hop neighbor to the left), while v forms a cage to its right (or v has no two-hop neighbor to the right). Recall that the cages are never destroyed. In this case u has *bridge right* while v has *bridge left* enabled. When either action is executed the predicate is satisfied. □

Lemma 6. *Assuming that neighbor relation at level $i-1$ does not change throughout the computation and* **good_links**.i, **one_links**.i, **zero_right_links**.i *as well as* **zero_left_links**.i *are satisfied, s-Tiara stabilizes to* **only_good_links**.i

Proof. (outline) The satisfaction of **good_links**.i, **one_links**.i, **zero_right_links**.i and **zero_left_links**.i leaves only one possible action en-

abled — *prune*. In this case there are links in $u.i.NB$ besides $u.i.rs$ and $u.i.ls$ and they are moved to $u.0.NB$. □

Lemma 7. *If a computation of Tiara starts from a state where $S(N)$ is connected, this computation contains a state where $B(N)$ is connected.*

Proof. The non-trivial case is where $S(N)$ is connected while $B(N)$ is not. That is, the overall graph connectivity is achieved through the links at the higher levels of Tiara. Let X and Y be two graph components of $B(N)$ such that they are connected in $S(N)$. Let $i > 0$ be the lowest level where X and Y are connected. Assume, without loss of generality that there is a pair of processes $a \in X$ and $b \in Y$, such that $a.i.rs = b$. In this case *downgrade right* is enabled at a. The execution of *downgrade right* connects X and Y in $B(N)$. The lemma follows. □

Define

$$\begin{aligned}\mathcal{SI} \;\equiv\; & (\forall i : i > 0 : \mathbf{good\_links}.i \wedge \mathbf{one\_links}.i \wedge \\ & \mathbf{zero\_right\_links}.i \wedge \mathbf{zero\_left\_links}.i \wedge \mathbf{only\_good\_links}.i)\end{aligned}$$

Lemma 8. *Tiara stabilizes to $\mathcal{SI}$.*

Proof. According to Lemma 7, every computation contains a state where $B(N)$ is connected. Due to Lemma 2, if $B(N)$ is connected, b-Tiara stabilizes to $\mathcal{GI}$. The remainder of the proof is by induction on the levels of s-Tiara. If $B(N)$ is connected and $\mathcal{GI}$ is satisfied, the consequent processes are linked. Hence, the right and left neighbor of each process at the bottom level does not change. Also, the five predicates above are vacuously satisfied for 0. Assume that these predicates are satisfied for all levels $\leq i-1$. Once the predicates are satisfied, none of the actions for processes at level $i-1$ are enabled. This means that the topology at this level does not change. Applying Lemmas 3, 4 5 and 6 in sequence we establish that the five predicates are satisfied at level i. Hence the lemma. □

3.3. Stabilization of Trim in b-Tiara

Link (a, b) is *independent* if there exists no link (c, d) different from (a, b) such that $c \leq a$ and $b \leq d$. Consider an arrangement where the nodes are positioned in the increasing order of their identifiers.

Lemma 9. *If a computation of b-Tiara that starts in a state where the graph is connected and contains an independent link of non-zero length, this computation also contains a suffix of states none of which contains this link.*

Proof. Let (a, b) be an independent link of non-zero length. None of the *grow* actions create independent links. The only action that makes a link independent is a *trim* of another independent link. Thus, if an independent link is deleted, it is never added again. Hence, to prove the lemma it is sufficient to show that (a, b) is eventually deleted.

Link (a, b) is non-zero length. This means that the node c consequent to a is not the same as b. In other words $a < c < b$. b-Tiara stabilizes to $\mathcal{GI}$ which ensures that a and c are connected. If c and b are not connected, both of them have a *grow* action enabled that connects them. Observe that (a, b) is independent. This means that all the right neighbors of a are to the left of b and all the left neighbors of b are to the right of a. Moreover, we just showed that there exists a node c such that $a < c < b$ and there are links $c \in a.R$ and $c \in b.L$. This means that *trim right* is enabled at a and *trim left* is enabled at b. The execution of either action deletes (a, b). □

We define the following predicate:

$$\mathcal{TI} \equiv (\forall a, b : a, b \in N : (a, b) \in E(B(N)) \Rightarrow \mathbf{cnsq}(a, b))$$

Lemma 10. *If Tiara starts in a state where it satisfies $\mathcal{GI}$ and $\mathcal{SI}$, then it stabilizes to $\mathcal{TI}$*

Proof. (outline) The conjunct of $\mathcal{GI}$ and $\mathcal{TI}$ is closed under the execution of b-Tiara. Note also that if $\mathcal{GI}$ and $\mathcal{SI}$ are satisfied, then the actions s-Tiara are disabled. Hence the closure of $\mathcal{TI}$.

Let us consider convergence. Since the actions of s-Tiara are disabled, they do not add links to $B(N)$. If $\mathcal{TI}$ does not hold, then there is at least one independent link of non-zero length. If the graph is connected, the *grow* actions never create an independent link. Consider a computation of b-Tiara that starts in an illegitimate state. Let ℓ be the length of the longest independent link. Since the state is not legitimate, $\ell > 0$. According to previous discussion, new links of length at least ℓ do not appear. Let (a, b) be the independent link of length ℓ. According to Lemma 9, (a, b) is eventually removed. Thus, all links of length ℓ are eventually removed. The lemma can be easily proven by induction on *ell*. □

The discussion in this section culminates in the following theorem.

Theorem 1. *Tiara stabilizes to the conjunction of $\mathcal{GI}$, $\mathcal{SI}$ and $\mathcal{TI}$.*

3.4. Stabilization Time

An *asynchronous round* is the shortest segment of the computation where for every process, if a guarded command is enabled at the beginning of the segment, this guarded command is either disabled or executed during the segment. We estimate the stabilization time of Tiara in such rounds.

Initially, the bottom level of Tiara may not be connected. Instead, the bottom level may be separated into components joined at some higher level. In this case, either *prune* or *downgrade* actions of s-Tiara are enabled at this level. Once such action is executed, the connecting link moves to the bottom level. Hence, the bottom level of Tiara is connected in $O(1)$ rounds.

Let us now consider the stabilization of the grow part of b-Tiara. As described in the convergence part of the proof of Lemma 2, once the bottom level of Tiara is connected, there is always a path between any two consequent processes a and b. Let us consider the shortest such path ρ. It must contain a segment d, e, f such that both d and f are either smaller than e or larger than e. Let us assume, without loss of generality, that both d and f are smaller than e. Let us further assume that e is the largest among the processes that belong to ρ. Observe that both d and f have enabled actions that eliminate e from the path between a and b. Moreover, since e is the extremum, it is never added to the path again. Therefore, any pair of consequent processes a and b is linked in no more than $|N| - 2$ rounds (see also [42] for a formal proof that uses the same argument for the synchronous case). That is, the grow part of b-Tiara stabilizes in $O(|N|)$ rounds.

Let us consider stabilization time of s-Tiara. According to Lemma 4, once level $i-1$ stabilizes, a process at level i goes up or down at most once. If the process at level i is finally up, the only actions that influence the stabilization of level i are *bridge* and *prune*. Once they are executed, level i stabilizes as well. That is, the number of stabilization steps at each level is proportional to the number of processes at this level. Since s-Tiara does not upgrade at least a third of processes from level $i-1$ to level i, the number of levels is at most $2 \log |N|$. Hence, the number of stabilization rounds can be estimated as follows:

$$\sum_{i=0}^{2\log|N|} |N| \cdot \left(\frac{2}{3}\right)^i < |N| \sum_{i=0}^{\infty} \left(\frac{2}{3}\right)^i = 3|N| = O(|N|)$$

Let us now estimate stabilization of the trim part of b-Tiara. After s-Tiara stabilizes, no links are added to the bottom level by s-Tiara. According

to the proof of Lemma 9, if there is an independent link, the trim actions that remove it are enabled. Therefore, if ℓ is the length of the longest independent link, all links of this length are trimmed in a single round. Hence, it takes at most $|N| - 1 = O(|N|)$ rounds for the trim part of b-Tiara to stabilize.

To summarize, the stabilization of every part of core Tiara takes at most $O(|N|)$ asynchronous rounds. That is, core Tiara stabilizes in $O(|N|)$ rounds.

4. Extension to a Ring (r-Tiara)

Outline. Tiara can be extended to a ring structure similar to Chord [9]. We call this extension r-Tiara. The idea is as follows. For b-Tiara, as well as for each level of s-Tiara, the lowest id-process needs to add a wraparound link to the highest-id process. This wraparound link maintenance is carried out by the process without left neighbors. After b-Tiara and s-Tiara stabilize, the lowest-id process at each level is the only such process. The highest-id process at each level is the only process without right neighbors.

Once the process determines that it has no left neighbors, i.e. all its neighbors have higher ids, it starts positioning the wraparound link. Essentially, the process continues to move the link to a right neighbor of the destination of the link. This movement stops once the wraparound link reaches the highest-id process at that level. If the maintainer of the wraparound link determines that it has left neighbors, it destroys its wraparound link.

Description. To simplify the presentation, the describe r-Tiara at the bottom level. In practice, r-Tiara is run at ever level of the skip list. This ring is established by connecting the lowest-id process on the level to the highest-id process on the level, which is respectively $min(N)$ and $max(N)$ on the bottom level. The code for r-Tiara is shown in Figure 8. In addition to the neighbor set, r-Tiara maintains a wraparound set ($u.0.WA$). Each process u has five actions: *wrap*, *extend*, *purge*, *expunge* and *reconnect*. Action *wrap* is enabled at the lowest-id process. Specifically, *wrap* is enabled if u does not have left neighbors, has a right neighbor, and currently is not participating in a wraparound link. In this case, u adds the rightmost process it has in its neighborhood to a wraparound variable. Action *extend* grows the wraparound link towards $max(N)$ on the bottom level. This action adds to the wraparound set a neighbor of u or a neighbor of a neighbor of u that has an identifier greater than any process id already in the wraparound set. When superfluous links exist in the wraparound variable, they are removed

process u
variables
$u.0.NB$, set of neighbor processes of u.
$u.0.WA$, wraparound link set
shortcuts
$u.0.L \equiv \{s : s \in u.0.NB : s < u\},\ u.0.R \equiv \{s : s \in u.0.NB : s > u\}$
actions

wrap: $(u.0.L = \varnothing) \wedge (u.0.WA = \varnothing) \wedge (u.0.R \neq \varnothing) \longrightarrow$
$u.0.WA := \{s : s \in u.0.R : (\forall z \in u.0.R : z \leq s)\}$

extend: $(u.0.L = \varnothing) \wedge (t \in u.0.WA \cup u.0.R) \wedge$
$(s \in t.0.R \cup u.0.R) \wedge (\forall z : z \in u.0.WA : s > z) \longrightarrow$
$u.0.WA := u.0.WA \cup \{s\}$

purge: $(u.0.L = \varnothing) \wedge (s \in u.0.WA) \wedge (\exists z : z \in u.0.WA : u < s < z) \longrightarrow$
$u.0.NB := u.0.NB \cup \{s\},$
$u.0.WA := u.0.WA \setminus \{s\}$

expunge: $(u.0.L \neq \varnothing) \wedge (s \in u.0.WA) \wedge (s > u) \longrightarrow$
$u.0.NB := u.0.NB \cup \{s\},$
$u.0.WA := u.0.WA \setminus \{s\}$

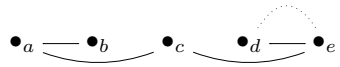
reconnect: $(u.0.R = \varnothing) \wedge (u \in s.0.WA) \wedge (s.0.L = \varnothing) \wedge$
$(\exists z : u \in z.0.WA : (z.0.L = \varnothing) \wedge (z > s)) \longrightarrow$
$u.0.NB := u.0.NB \cup \{s\},$
$s.0.WA := s.0.WA \setminus \{u\}$

Figure 8: The ring component of Tiara (r-Tiara).

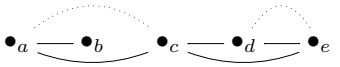
by the *purge* action. In the case where u gains a left neighbor and is currently maintaining the wraparound variable, action *expunge* removes all links to the right of u from the wraparound variable. Action *reconnect* handles a rather curious case where the only connectivity between two graph components is through the wraparound link. Let the two components be S and Z such that s is the rightmost (lowest id) process in S and z is the rightmost process in Z. Both s and z are connected to process u via a wraparound link and this link is the only connectivity between S and Z. In this case, the link is removed and placed in the neighborhood. Actions *purge*, *expunge*, and *reconnect* move links to the neighbor variable to prevent the possibility of partitioning the graph. The operation of r-Tiara on the bottom level is illustrated in Figure 9.

Correctness proof. We denote by $R(N)$ the graph induced by the processes, the b-Tiara links (the level 0 s-Tiara links) and the wraparound links. We define the following predicate

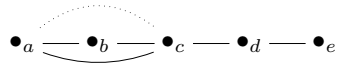
$$\begin{aligned} \mathcal{RI} \ \equiv \ & (min(N).0.WA = max(N)) \ \wedge \\ & (\forall a : a \in N \setminus \{min(N), max(N)\} : a.0.WA = \varnothing) \end{aligned}$$



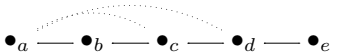
(a) *wrap* is enabled at a and *grow right* is enabled at c. The wraparound link (a, c) is established and link (c, d) is added.



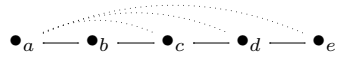
(b) *expunge* is enabled at d, *trim right* is enabled at c, *trim left* at e and *grow left* is enabled at c. The wraparound link (d, e) is moved to $d.0.NB$. Link (c, e) is removed. Link (b, c) is added.



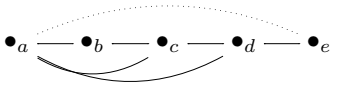
(c) *extend* is enabled at a, *trim right* is enabled at a and *trim left* is enabled at c. The wraparound link is extended to d and link (a, c) is removed.



(d) *extend* is enabled at a, which extends the wraparound link to e.



(e) *purge* is enabled at a. Links (a, c) and (a, d) are moved to $a.0.NB$.



(f) The system has reached a legitimate state with only one wraparound link and $min(N).0.WA = max(N)$.

Figure 9: Example computation of *r-Tiara*. The processes are listed in increasing order of their identifiers. Not all enabled actions are listed.

That is, $\mathcal{RI}$ states that the only wraparound link that the bottom level has connects the nodes with the largest and smallest identifiers.

Lemma 11. *If a computation of b-Tiara, s-Tiara and r-Tiara starts from a state where $B(N)$ is connected, it stabilizes to $\mathcal{GI} \wedge \mathcal{SI} \wedge \mathcal{RI}$.*

Proof. Let us address the closure first. We have to consider the influence of the actions of r-Tiara on $\mathcal{GI}$ since some of them modify the variables of b-Tiara. Fortunately, *purge* and *expunge* are the only such actions. Moreover, they cannot invalidate $\mathcal{GI}$ since they only add links to the neighborhood of a node and do not remove them. The actions of r-Tiara do not influence $\mathcal{SI}$. Let us attend to the closure of $\mathcal{RI}$. If $\mathcal{GI} \wedge \mathcal{SI} \wedge \mathcal{RI}$ is satisfied, the only node on the bottom level that has an empty left neighborhood is $a = min(N)$. Thus, only a may have *wrap* and *extend* enabled. Yet, $\mathcal{RI}$ stipulates that the wraparound variable of a is not empty. Thus, *wrap* is disabled when $\mathcal{RI}$ holds. Also if $\mathcal{RI}$ holds, the wraparound link connects a and $b = max(N)$. Node b has an empty right neighborhood ($b.0.R$). Thus, *extend* is also disabled at a. Let us discuss *expunge*. If $\mathcal{RI}$ holds, b is the only node whose left neighborhood is not empty yet $b.0.WA \neq \varnothing$. However, for b, $b.0.WA = a$ which is less than b. Hence, *expunge* is disabled as well. Action *purge* is disabled by definition if $\mathcal{RI}$ holds.

Let us now address the convergence of the predicates. r-Tiara does not affect the convergence of $\mathcal{GI}$ or $\mathcal{SI}$ as it can only add links to a node's neighborhood but not remove them. Thus, we can prove the convergence of $\mathcal{RI}$ while $\mathcal{GI} \wedge \mathcal{SI}$ is satisfied. If $a = min(N)$ on the bottom level does not have a wraparound link, *wrap* is enabled. The execution of *wrap* creates this link. No other wraparound links can be created. That is, if a computation starts from a state where a has a wraparound link, the number of wraparound links in the states of this computation can only decrease. Consider a wraparound link whose left (smaller) incident link is not a. *expunge* is enabled at this node. The execution of *expunge* removes such link. That is, each computation contains a suffix where a is incident to the only existing wraparound link. If $b = max(N)$ on the bottom level is not incident to this link, *extend* is enabled at a. If *extend* is executed the identifier of the right incident node increases. Thus, the computation contains a state where b is incident to this link. If $b \in a.0.WA$ and $a.0.WA \neq \{b\}$, then *purge* is enabled at a. When executed only b will remain in $a.0.WA$.

That is, if $\mathcal{GI} \wedge \mathcal{SI}$ is satisfied, a computation of r-Tiara contains a suffix where in each state there is a single wraparound link that connects processes

with the minimum and maximum ids on the level. That is r-Tiara converges to $\mathcal{RI}$. □

Lemma 12. *If a computation of b-Tiara, s-Tiara and r-Tiara starts from a state where $R(N)$ is connected, this computation contains a state where $B(N)$ and $S(N)$ are connected.*

Proof. (outline) The only non-trivial case is where the computation of b-Tiara, s-Tiara and r-Tiara start from a state in which $R(N)$ is connected while $B(N)$ and $S(N)$ are not. That is, the overall graph connectivity is achieved via wraparound links.

Let X and Y be two disconnected graph components of $B(N)$ and $S(N)$ such that there is a pair of processes $a \in X$ and $b \in Y$ and $a.0.WA = b$. Let us consider each component as a separate system. According to Lemma 11, r-Tiara arrives at a state where the only existing wraparound link connects the processes with the smallest and largest identifiers in each component. That is link (a, b) is no longer a wraparound link. r-Tiara does not delete the wraparound links. Instead, it moves them to $B(N)$. Thus, the computation contains a state where the (a, b) link belongs to $B(N)$. That is, the graph components X and Y are connected. The lemma follows. □

Lemmas 11 and 12 combined yield the following lemma

Lemma 13. *If a computation of b-Tiara and s-Tiara with r-Tiara starts from a state where $R(N)$ is connected, it stabilizes to $\mathcal{GI} \wedge \mathcal{SI} \wedge \mathcal{RI}$.*

5. Extension to Skip Graph (g-Tiara)

Outline. The disadvantage of using skip lists for peer-to-peer systems is their low expansion. For example, if the skip list does not have the ring extension, a failure of a single node can disconnect it. To mitigate this problem we propose to extend Tiara to concurrently construct multiple skip lists forming a skip graph. We call this component g-Tiara.

The idea is as follows. Recall that at each level $i - 1$, the core Tiara does not upgrade at least one-third of the nodes. The upgraded nodes form a list at level i while the remainder do not form any links in higher levels. In g-Tiara, the remaining nodes form their own list at level i. This list is used to concurrently run the next level of Tiara. In our description we ignore the special cases that arise at the edges of the skip graph because they turn out to be inconsequential. We also ignore the operation of r-Tiara at each level.

g-Tiara does not interfere with the operation of s-Tiara but uses the nodes at level $i-1$ that are not upgraded by s-Tiara to construct an alternative list of nodes i' at level i. Similarly, at the next level $i+1$, s-Tiara and g-Tiara construct main and alternative lists as well. Moreover, once the alternative list is built at level i, a separate instance of s-Tiara and g-Tiara operates on it to construct a pair of lists at level $i+1$. In other words, there are $\log_2 i$ lists at level i of a skip graph. Refer to Figure 1 for an illustration of a complete skip graph built by this extension to core Tiara.

Description. The code of g-Tiara is shown in Figure 10. We assume that b-Tiara is in operation. That is, b-Tiara links the processes with consecutive identifiers at level 0. At each level $i > 0$, s-Tiara constructs a list of nodes upgraded from level $i-1$. If the process is not upgraded, it is *down*. Specifically, s-Tiara constructs an *0-1* sparse skip list. In this skip list, at most two consecutive processes are upgraded to the next level. Therefore, a down process at level $i-1$ can be at most three hops away from another down process.

The idea of g-Tiara is to locate down processes within three hops. For that, each process u at level i, maintains a set $u.i.NB2$ of two-hop neighbors. This set is a copy of all the identifiers stored by the one-hop neighbors. The latter is referred to as $u.i.NB2DST$.

The alternative set of neighbors is collected in $u.i.NB'$. This set contains all the down processes that are at most three hops away at level $i-1$. The closest identifiers in $u.i.NB'$ are the left and right neighbors in the alternative list thus constructed by g-Tiara.

g-Tiara has two actions. Action *gather* maintains $u.i.NB2$ on the basis of the information stored in single-hop neighbors. Action *connect* updates $u.i.NB'$. Note that even though *gather* shows the construction of the neighborhood at level i, *connect* uses two-neighborhood information at level $i-1$.

To avoid partitioning, the incorrect links are moved to $u.0.NB$. Note that according to the execution semantics, once a process z is added to one of the sets maintained by u, for example $u.i.NB'$, process u is also added to the equivalent set in z, that is $z.i.NB'$.

Let us examine the neighborhood $u.i.NB$ that is formed by each node u upgraded by s-Tiara. The links to the closest left and right neighbors of u form a list of nodes at level i. This list is used to construct $u.(i+1).NB$ by s-Tiara and $u.(i+1).NB'$ by g-Tiara. Similarly, on the basis of $u.i.NB'$, an alternative list of nodes not upgraded by s-Tiara is formed.

process u
parameter $i \geq 0$: **integer** — level of the skip graph
constant $u.i.NB$ — set of neighbor processes of u at level i
variables
$u.i.NB2$ — 2-hop neighborhood set variable of process u at level i
$u.i.NB'$ — alternative neighbors of u
shortcuts
exists$(z, i) \equiv ((z \neq \perp) \wedge (z.i.NB \neq \varnothing))$
$u.i.NB2DST \equiv \{s : (\forall t \in u.i.NB : s \in (u.i.NB \cup t.i.NB))\}$
$u.i.DOWN \equiv \{z : z \in \{s : (\forall t \in u.(i-1).NB2 :$
$s \in (u.(i-1).NB2 \cup t.(i-1).NB)\} : \neg$**exists**$(z, i)\}$
actions
for $i \geq 0$
gather: $u.i.NB2 \neq u.i.NB2DST \longrightarrow$
$u.0.NB := u.0.NB \cup u.i.NB2 \setminus u.i.NB2DST$,
$u.i.NB2 := u.i.NB2DST$
for $i > 0$
connect: $u.i.NB' \neq u.i.DOWN \longrightarrow$
$u.0.NB := u.0.NB \cup u.i.NB' \setminus u.i.DOWN$,
$u.i.NB' := u.i.DOWN$

Figure 10: The skip graph component of Tiara (g-Tiara).

Correctness proof. The operation of the algorithm is rather straightforward. We, therefore, present the informal correctness statement in the below theorem and show the proof outline.

Theorem 2. *At each level i, for each down process u, the nearest left and right down neighbors of u are in the alternative neighborhood set $u.i.NB'$.*

Proof. (outline) The proof is by induction. The bottom level is maintained by b-Tiara. This level stabilizes to $\mathcal{GI}$ regardless of actions of g-Tiara. Thus, the list of the processes at the bottom level eventually remains unchanged. This allows r-Tiara to stabilize to $\mathcal{RI}$ at level 0. Assume that the list of processes and the corresponding wraparound link does not change at level $i-1$. The construction of s-Tiara list at level i proceeds independently of g-Tiara. After s-Tiara stabilizes to $\mathcal{SI}$, the list at level i is a sparse $0-1$ skip list. This means that for each down node the nearest neighbor is at most three hops away.

Meanwhile, due to action *gather* of g-Tiara, for each process u, $u.(i-1).NB2$ contains correct two hop neighborhood information. After this information is stable, *connect* adds down processes up to three hops away from u to $u.i.NB'$. The theorem follows. □

Complexity estimates and expansion bounds. The skip graph can be viewed as a collection of skip-lists rooted in every node. Thus, unlike a skip list, the search in a skip graph has a downward phase only. Therefore, the number of steps required to perform a search in a skip graph is in $O(\log |N|)$.

In a *random routing problem*, every node in the network has exactly one message for a node chosen uniformly at random [3, 57]. Given a routing strategy R in some graph G, the *congestion* of a fixed routing problem is the maximum number of messages traversing a node when using R for that problem. When using a random routing problem, we are interested in the expected congestion. In a complete binary tree, for example, the expected congestion is $\Theta(|N|)$ as, on expectation, half of the nodes below the left son of the root want to send their message to one of the nodes below the right son of the root, and vice versa. Hence, $\Theta(|N|)$ messages have to cross the root on expectation, no matter which routing strategy is used for them. The same bound holds for the skip list since the removal of the root node in the skip list cuts it into two connected components of approximately the same size. Thus, any message from one of these components to the other has to cross the root node. A much better congestion can be achieved for the skip graph.

In our g-Tiara skip graph, any edge in a level i-ring connects nodes of distance at most 3 in the $(i-1)$-ring and no three consecutive nodes belong to the same i-ring. Hence, any edge of an i-ring can skip at most 3^i nodes on the base ring, and any i-ring contains at most $(2/3)^i \cdot |N|$ nodes. Consider the strategy of routing every request to its destination in a top-down fashion: in each hop, the highest level edge towards the destination is used without getting beyond the destination. Then every request traversing an i-level edge must originate from a node in that i-ring. Therefore, the expected number of requests passing a node at level i is at most

$$\left(\frac{2}{3}\right)^i \cdot |N| \cdot \frac{3^i}{|N|} = 2^i$$

We can only have that many requests if $2^i \leq (2/3)^i \cdot |N|$ (the number of nodes in level i), which is true so long as $i \leq \log_3 |N|$. Therefore, the expected congestion is at most $2^{\log_3 |N|} = |N|^{1/\log 3}$.

With the help of the expected congestion, we can find a lower bound on the expansion of a graph. Let U be an arbitrary subset of nodes in the given

graph. A neighbor set $NB(U)$ is defined as $\{w \in V \setminus U \mid \exists v \in U : (v, w) \in E\}$. That is, a node belongs to $NB(U)$ if it does not belong to U but is a neighbor of at least one node of U. The *(node) expansion* of a graph is defined as $\min_{U \subset N, |U| \leq |N|/2} |NB(U)|/|U|$. The expansion captures the degree of connectivity of a graph, and the best expansion of a constant degree graph is constant.

Theorem 3. *The expansion of g-Tiara is* $\Omega(1/|N|^{1/\log 3})$.

Proof. Given an expansion of α, it is possible to design a permutation routing problem (every node is the source and destination of exactly one message) so that there is a node that is passed by at least $1/\alpha$ many messages, no matter which routing strategy is used (all messages from the set U with $|NB(U)| = \alpha|U|$ are requested to leave U). On the other hand, every permutation routing problem can be solved via two random routing problems: first, route each message to a random intermediate destination and then from there to its final destination. In fact, when using this strategy, the expected congestion for routing any permutation routing problem is twice the expected congestion of a random routing problem (which was first observed by Valiant [58]). Thus, if a random routing problem can be routed with expected congestion C, any permutation routing problem can be routed with expected congestion $2C$. Using the well-known Markov inequality, this implies that for any permutation routing problem there is a routing strategy with congestion at most $3C$. This, however, is only possible if the expansion of the given graph is at least $1/3C$. Now, using the fact that the expected congestion for a random routing problem in g-Tiara is at most $|N|^{1/\log 3}$, the theorem follows. □

Thus, expansion of g-Tiara is much better than the expansion $\Theta(1/|N|)$ of the tree and the skip list. When using a randomized skip graph as proposed in [39], even a constant expansion can be reached [59], but this type of a skip graph requires a suitable extension to be locally self-stabilizing [47].

6. Implementation and Further Extensions

Searches. Core Tiara maintains a skip list which is equivalent to a distributed balanced search tree. The searches in skip lists are proceed similar to searches in such trees.

Let b be a right neighbor of a at some level i of Tiara. The *right interval* of a, denoted $[a, b)$, is the range of identifiers between a and b. *Left interval* is defined similarly. If a does not have a right neighbor, its interval is not finite. That is, a's interval contains all process identifiers greater than a. Similarly, if a lacks left neighbor it's interval is infinite on the left. Thus in any level, the collection of intervals contains the complete range of identifiers.

Suppose a, c and b are consequent at level $i-1$ of Tiara and a and b are consequent at level i. That is, c is in the cage. Since the identifiers are sorted, c belongs to the interval $[a, b)$. If a node is down, then one if its neighbors is up. Thus, a client process that has a pointer to a node in Tiara and wishing to advance up the skip list only needs to examine the node's neighbors.

Assuming that a client process connects to an arbitrary node in Tiara, the search proceeds first upward then downward in the skip list. In the upward phase, the client is moving up the list looking for the node whose interval contains the identity. Since every level contains the complete id-range, this phase terminates. Once the range is found, the client advances downward evaluating the cages it encounters to narrow the search range. This procedure continues until the desired node x is located or it is established that x belongs to the interval of the consequent nodes at the bottom level. The latter case means that x is not present in the system. There are $O(\log |N|)$ levels in Tiara. Thus, the upward and the downward phases take $O(\log |N|)$ number of steps.

Joins and leaves. We describe how core Tiara as well as g-Tiara can be expanded to incorporate voluntary node joins and leaves. Crashes are not considered. We only discuss single join or leave. Handling concurrent joins and leaves, i.e. churn, is left for future research.

We cover core Tiara first. We assume that each process has two read-only Boolean variables maintained by the environment: *join* and *leave*. Since the variables are read-only, stabilization of their operation is the responsibility of the environment. Let us consider the join operation first. The joining node x connects to an arbitrary node of the network. The variable *join* is set to **true**. We assume that the environment may only set *join* to **false** after the node successfully inserts itself at the bottom level of Tiara. The joining node executes a search to find the bottom level interval $[a, b)$ to which it belongs. Then, x makes a and b its right and left neighbors respectively. After a and b discover the presence of a node u whose *join* is set to **true**, they remove link (a, b) (while maintaining their links to u). Then, the upper levels of

Tiara adjust. The insertion of the node at the bottom level entails at most a constant number of steps at each level of Tiara. Since the search takes at most $O(\log |N|)$ steps, the total number of steps required for node join is also in $O(\log |N|)$.

Let us discuss the leave operation. The environment sets *leave* to **true** to indicate that the node x requests disconnect. We assume that *leave* cannot be set when *join* is set and it cannot be set back to **false** until the node disconnects. When the right and left neighbors of x notice that the *leave* of x is set to **true**, the neighbors add a link bypassing x at the bottom level. Node x can then disconnect. The higher levels of Tiara execute the regular Tiara actions to accommodate the missing node. At most a constant number of adjustment steps is required at each level. Hence the total number of steps required for the node to leave Tiara is in $O(\log |N|)$.

Let us now estimate the complexity of these operations in g-Tiara.

Lemma 14. *A join of a node at level $i-1$ requires a constant number of steps, a join of a node in one of the resultant lists at level i and possibly a change of a single node in the other list. Similarly, a leave of a node at level $i-1$ requires a constant number of steps, a leave of a node in one of the resultant lists at level i and a change of a single node in the other list.*

Proof. We discuss node join. The argument for node leave is similar. Let node z join the neighborhood of node u at level $i-1$. Let the neighbor aliases be as shown in Figure 4.

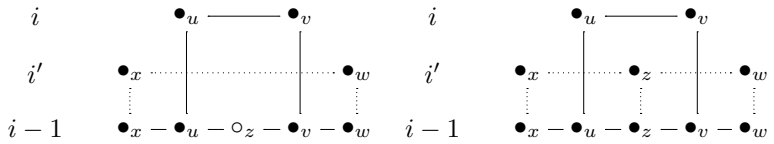
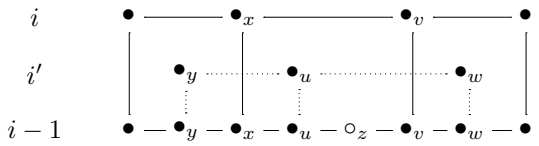
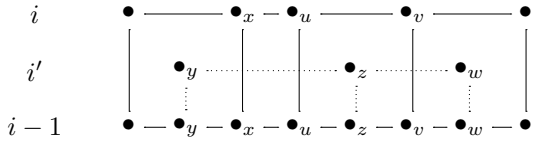


Figure 11: Node z joins the skip graph at level $i-1$ under the bridge of the s-Tiara list.

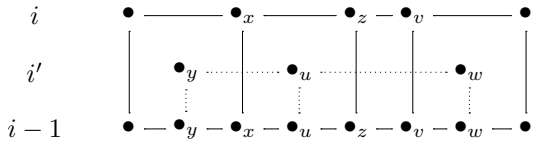
A node may join under the bridge of s-Tiara or inside a cage. Let us consider the bridge first. See Figure 11 for illustration. In this case, the actions of s-Tiara are disabled and the only outcome is for z to be added to the list maintained by g-Tiara at level i'. This requires two actions of g-Tiara. That is, one of the sublists does not change while a node joins the other list.



(a) initial state



(b) final state if u or v executes the *upgrade* action

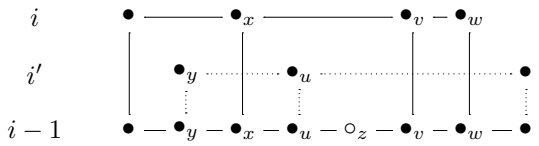


(c) final state if z or x executes *upgrade*

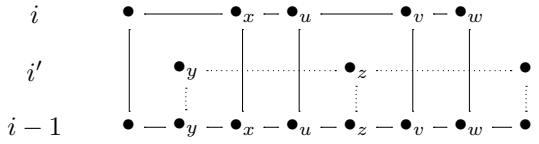
Figure 12: Node z joins the skip graph inside a cage with no adjacent bridges.

Let us now consider the case of z joining inside a cage of s-Tiara. The outcome differs depending on whether there are bridges adjacent to this cage. We discuss the subcase of no adjacent bridges. Refer to Figure 12. The appearance of z enables actions of s-Tiara at x, u, z and v. The execution of *downgrade* actions at x or v does not affect the final state. The final state depends on which process executes an *upgrade* action. If process u or v executes *upgrade*, u joins the s-Tiara list at level i while z replaces u in g-Tiara list. If process x or z executes *upgrade*, z joins the s-Tiara list while, g-Tiara list is not affected. In either case the conditions of the lemma are satisfied.

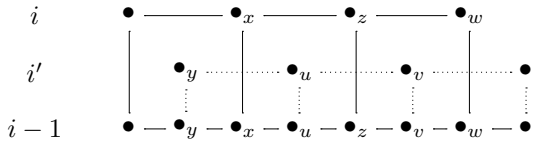
The last case to consider is where z joins a cage and there is an adjacent bridge. We only discuss the case where there is a single adjacent bridge to the right of the cage. The other cases are similar. Refer to Figure 13. Again, the final state depends on which process executes *upgrade*. If u or v execute this action, it results in the formation of the second bridge, u joining the s-Tiara list and z replacing u in the g-Tiara list. However, if z or x execute *upgrade*, three consequent nodes z, v and w are upgraded to level i from level $i-1$ in s-Tiara. This forces v to execute *downgrade center*. The final state results in v joining the g-Tiara list while z replacing v in s-Tiara. This



(a) initial state



(b) final state if u or v executes *upgrade* action



(c) final state if z or x executes *upgrade*

Figure 13: Node z joins the skip graph inside a cage with adjacent bridge to the right.

satisfies the conditions of the lemma. □

We calculate the complexity of a topology update based on Lemma 14. According to the lemma, the topology update requires a constant number of steps and results in a similar topology update in one of the sublists and a node replacement in the other. Let c be the constant upper bound on the number of steps required for this propagation to occur.

Single member replacement then requires c number of steps at each level in one of the sublists. In other words, the propagation of the update to a single level i results in the addition of $i \cdot c$ steps. There are at most $\log |N|$ levels in the skip graph. Thus, the total number of steps required to add or remove a node to the g-Tiara skip graph is:

$$\sum_{i=0}^{\log |N|} i \cdot c = c\frac{\log |N| \cdot (\log |N| + 1)}{2} = O(\log^2 |N|)$$

Note that the described implementation of topological updates requires participation of the environment. Fully implementing the updates as a part of the self-stabilizing algorithm is an interesting direction for future Tiara extension.

Other improvements. There are a number of modifications to Tiara that make it more efficient and applicable to practice. At each level of Tiara, up to two out of three nodes may be promoted to the next level. Although the number of levels is logarithmic with respect to the system size, it may still be relatively large. The number of levels may be decreased by modifying Tiara to promote fewer nodes. For example, we can allow the nodes at level i to skip up to two or three neighbors at level $i-1$. This would require for each node to maintain data about its extended neighborhood.

The *grow* operation of b-Tiara may force a process to acquire up to $O(|N|)$ neighbors during stabilization. This may require devoting extensive memory resources of each node to neighborhood maintenance. A simple way to mitigate it is to execute *trim* operations before *grow*. That is, if a process finds that it has both *trim* and *grow* actions enabled. It executes *trim*. Care must be taken to ensure that action execution is still weakly fair.

7. Future Work

We presented Tiara — the first deterministic self-stabilizing peer-to-peer system with a logarithmic diameter. It provides a blueprint for a realistic system. An important further task is to study the implementation of Tiara in more realistic low-atomicity models such as message passing. We envision two approaches to this. We can refine the atomicity of Tiara using, for example, self-stabilizing dining philosophers implementation [60]. In this approach, only one neighbor is allowed to modify its state and links at a time. Alternatively, Tiara may be redesigned to allow non-interfering concurrent topology updates. The latter idea may prove to be a lot more challenging but with a greater potential performance gain. Recently, this approach yielded an efficient stabilizing skip list construction in message-passing [61].

We envision several other directions of extending this work: further efficiency improvements, such as keeping the runtime and the degree of the self-stabilization process low, and adding features required by practical systems. An important property is resistance to *churn* — continuous leaving and joining of nodes. As evidenced by a number of studies [62, 63] dealing with churn in peer-to-peer systems is rather complicated. Formally addressing churn along the lines described by Li et al [64] would be an interesting avenue of further research. Several studies (cf. [4]) present non-stabilizing deterministic structures with better expansion and congestion properties than Tiara.

In case randomization is allowed, constant expansion is achievable [59]. In the future, it would be interesting to investigate a stabilizing structure that improves these properties over Tiara.

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